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An Under Water Sound of Natural Origin

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(Received October 1, 1942)

DURING the past dozen years we have made many voyages, short and long, in our small thirty-foot ketch over the waters of the Atlantic coast from North Carolina to Maine. Throughout this time we have noticed on five occasions certain water-borne sounds which originated from natural causes and not from man-made sources. The sounds were similar in type, and although of moderate or low intensity, impressed the attention immediately as out of the ordinary and foreign to our usual experience.

The first occurrence was at the entrance of Chesapeake Bay several miles to the westward of the line between the lighthouses of Cape Henry and Cape Charles. It was after sunset of a day in July and the little boat was headed out to sea. A three-day spell of northeast weather had come to an end, the breeze had fallen to a light air, almost to the calm that presaged the setting in of fair southerly weather. A long swell lifted us lazily up and down and the surface of the sea was without ripples. As we ghosted gently along, close-hauled on the port tack, for the light air still persisted in its northeasterly direction, we became conscious of a slight sound that gradually increased in intensity as we progressed on our course to the eastward. At first the sound was barely audible from the cockpit, below decks it was easily heard. There, and better still to one closed up in the small engine compartment or with ear pressed to the outside planking, the sound was very plain. It was of relatively high pitch and was made up of many little impulses. We spoke of it as a "crackling" sound. We were unable to determine that the sound came from any particular direction.

When putting out to sea at night in a small boat one is concerned about little things. We imagined a leak or that we were dragging a blackberry bush under the keel, but there was no leak and no blackberry bush. We strapped the sails in tight and turned the vessel into the wind until she practically ceased to move through the water, but this had no effect on the intensity of the sound. As we proceeded on our course the sound grew louder until it was easily heard on deck and the separate "crackles" were clearly noticeable. We imagined that it might come from pebbles on a beach rolled back and forth by surf, but the nearest beach was six miles away and there certainly were no pebbles rolling about beneath us.

Red lines on the chart indicated a cable area

about four miles ahead. We wondered if somehow or other an electrical influence from the cables might introduce a sound into the water. However, in an hour we had neared the cable area and meanwhile the sound had reached a maximum and had begun to die away. In another hour we had passed the area and the sound had practically disappeared. Some weeks later the suggestion was offered that when we heard the sound we were approximately at the center of the arc of the Cape Henry shore, near Lynnhaven Roads, and at the focus of the water-borne sound of the surf there. The explanation seemed adequate but subsequent experiences threw doubt on its correctness.

The second occurrence of the sound was three years later off Cape Hatteras. Again it was July and the conditions were much the same as in the case near the Virginia Capes. Diamond Shoal Light Ship lay several miles behind us to the south; we were headed north before a faint southerly air making barely perceptible progress over an almost mirror calm sea. About ten o'clock in the evening the sound set in very faintly and continued for about two hours, rising to a maximum and diminishing again. The sound again was of the "crackling" type, similar to that of the first case as well as we could remember, but at no time was as loud, being audible in the cabin but not in the cockpit. We were thankful for the previous experience, for we were able immediately to recognize the sound, if not as an old friend, at least as a harmless acquaintance. However, the theory of focusing of the sound of surf on a beach that seemed reasonable in the first case, did not seem applicable in this case, because the nearest beach was approximately straight, parallel to our course and about fifteen miles away.

During August a year later we anchored in the harbor of Beaufort, North Carolina. Immediately after the sails were furled and quiet reigned, we heard the sound loudly, more loudly than we had ever heard it before. It continued steadily night and day during the two days we were there with no noticeable change in intensity. The separate "crackles" were much louder and perhaps more separated in time than in the cases at sea. They sounded exactly like hail on a window plane. There was a to and fro run of tide, sometimes the water was rippled by breezes and sometimes it was calm. But throughout these changes the sound continued without variation. We asked a fisherman if he had noticed the sound. He said that it was well-known there and was caused by the toadfish gnashing their teeth.

Last year at the schooner anchorage at Norfolk, Virginia, the sound was distinctly audible but not very loud. We heard it there one evening at the end of August and again when we stopped in the anchorage a week later. The following night at anchor off Mobjack, Chesapeake Bay, the sound was present very faintly. We had been at Norfolk and Mobjack several times in previous years but had never heard the sound in those places before.

The foregoing instances are a complete record of our experiences in hearing the sound. The sound was always louder below than above deck. It seemed probable, insofar as memory may be depended upon, that the sounds were of the same type, and hence of similar origin, in all the cases. The data do not permit one to infer that the sound is a rare or infrequent phenomenon. True, we have heard the sound only a few times but we have not remained long in any of the places where the sound was.

It would be interesting to know whether others have noticed the sound. It is probable that records of its occurrence in open water may not be extensive, because cases of observers at sea in very quiet boats may be expected to be the exception rather than the rule. We have never heard the sound north of the Chesapeake although we have been in many harbors from Cape May to Mount Desert, often when conditions were quiet, and have been at sea in gentle or calm weather in various localities along that portion of the Atlantic coast. We feel sure that we would have heard the sound if it had been present in northern waters. We favor the toadfish theory of the origin of the sound but wonder whether toadfish have teeth.

The Use of Counter Circuits in Frequency Dividers

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(Received October 24, 1942)

UNTIL recent years the circuits used most commonly as frequency dividers were oscillatory circuits having a natural period somewhat longer than that of the frequency desired at the output of the divider. The input wave, whose frequency was to be divided, acted to cause the oscillatory circuit to start a new cycle of operation after a given number of cycles of the original wave had elapsed. For example, if it is desired to divide a 600-cycle wave by 3 and produce a 200-cycle wave, the oscillatory circuit is designed to have a natural period somewhat longer than 1/200 second. During the first cycle of operation of the oscillatory circuit it is insensitive to the input wave until after 3 cycles of the 600-cycle wave have occurred and the fourth cycle trips the oscillatory circuit again so that the process repeats. Such divider circuits used gaseous discharge tubes, multibrator stages, or blocking oscillators.

In some applications these dividers were not satisfactory because of instability due to fluctuations in supply voltages, or due to changes in the tube characteristics. This was especially true if the divisor was large. They were particularly troublesome if the input frequency was not fixed, for if the input frequency varied an appreciable amount the divider circuit would become instable or would divide by a different amount than was desired.

The Electric Music Industries, Ltd., of Great Britain developed the pulse-counter circuit for synchronizing-signal generators and it was put into use in this country by RCA. Bedford and Smith describe the frequency divider using a counter circuit in the July, 1940, *RCA Review*.

This divider circuit is extremely stable for counts up to 16 and higher, and the input frequency can be varied over a 10 to 1 range or more without changing the count or divisor.

The counter circuit counts a given number of pulses without regard to their frequency and fires to produce a single pulse after the required number of pulses have occurred. The counting is accomplished by charging a condenser in a stair-step fashion as shown in Fig. 1.

The voltage across this condenser is applied to a trigger circuit that trips at a predetermined voltage level. Upon tripping the trigger circuit in a very brief interval of time discharges the condenser, creates an impulse which serves as the output voltage from the divider, and prepares itself to stand by until the condenser voltage again reaches the critical value. For every cycle of the input frequency one step is produced and the number of steps that will occur before the trigger circuit trips will depend upon the size of the risers on these steps and upon the critical voltage of the trigger circuit. The length of the tread on these steps is determined by the input frequency and has no influence on the action of the trigger circuit.

For best results the input wave to the counter circuit should be of rectangular wave form. This may be accomplished by a limiter tube V_1 shown in Fig. 2. The voltage input is large enough to drive the tube to cut-off in one direction and to saturation in the other direction. So the output of the limiter is of rectangular wave form and its peak to peak value is essentially equal to the $+B$ voltage. The voltage output from the limiter is applied to the series combination of the condensers C_1 and C_2, and the diode V_2. When the limiter output is changing in the positive direction, a charging current flows through this series combination. This current charges the condensers

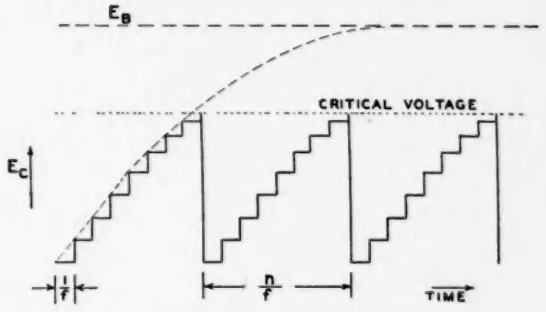

FIG. 1.

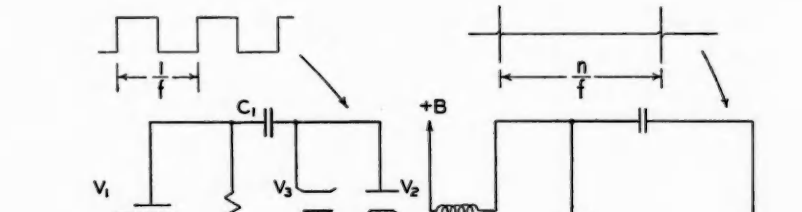

FIG. 2.

so that the peak voltage is divided between them inversely as their respective capacitances. C_1 is usually small compared to C_2 so that the voltage across C_2 will be small compared to the peak value of the applied signal. This action produces the riser on the first step in the stair-step voltage across C_2 shown in Fig. 1.

As was stated before, the number of steps that will occur before the trigger circuit trips depends upon the size of the risers on these steps and the critical voltage of the trigger circuit. The size of the risers will depend upon the peak to peak value of the rectangular voltage wave and the ratio of C_1 to C_2. The critical voltage of the trigger circuit depends upon the bias on the trigger tube. Therefore for a given limiter tube operation C_1 or C_2 may be adjusted to change the count or divisor of the circuit, or the bias on the trigger tube may be changed. In some cases it will be found desirable to make C_1 adjustable to give large changes in count and a bias adjustment can be used for minor adjustment or calibration.

The trigger tube shown in Fig. 2 is of the blocking oscillator type which has been used rather widely for sweep circuits in television. It might be of the multivibrator type, which according to Bedford and Smith is preferable in their television synchronizing-signal generator. The trigger tube might be of a gaseous type such as a 2050, but gaseous tubes are usually avoided where extreme stability is desired. We have found the blocking oscillator quite satisfactory from the standpoint of stability and simplicity.

The grid of V_4 in Fig. 2 has no d.c. path to ground except what may be present in the form of leakage resistance, so the trigger tube is not self-oscillatory except over long periods of time due to leakage resistance. The choice of value in C_2 will determine the time constant for the particular leakage resistance and will limit the low frequency output of the frequency divider. The period of oscillation may readily be in the order of 15 seconds and is of no particular concern except where very low frequencies are involved. If the time constant of this circuit is too low for the frequencies involved it will show up in the stair-step voltage wave across C_2 by a downward slant of the treads on the steps. The downward slope becomes increasingly large as the number of steps is increased so that there is a tendency toward instability for large counts as the net gain for a given step high on the stairs will be small because of a loss of voltage in the tread. This nearly equals or may exceed the rise in voltage on the riser. Such a condition is easily diagnosed with an oscilloscope. For best results the tread should be horizontal and the risers nearly vertical. In order that the risers may be nearly vertical we must consider the time constant of the circuit in which C_1 and C_2 are charged. The effective r.c. of this circuit should be so low that the time constant is negligible compared to the time of a period $1/f$. If this time constant is not negligibly small the riser takes on a curved shape, producing rounded steps instead of square steps and the critical voltage might be reached on the rounded portion of this riser to produce a trigger action at a variable time, depending upon certain variations in the circuit. The period of the output of the frequency divider would therefore be a

variable which was not strictly an integral submultiple of the input.

If the input frequency is to be extremely high serious attention must be given to the time constant. There are certain practical limitations that will be encountered in reducing r.c. to extremely small values. The resistor through which the condensers are charged and discharged cannot be so small that the voltage swing across it differs materially in magnitude from E_b. Also, there is a limit in the minimum sizes of C_1 and C_2. If C_1 is made quite small and C_2 is reduced accordingly to keep the proper ratio between these capacitances, the shunting capacity between the diode elements and ground and other wiring involved may be appreciable compared to C_2 and in that way limit the reduction of C_1. However, with proper design considerations such as are now common in video amplifier design it is possible to operate such a counter with an input frequency of several hundred kilocycles. If the divider is to be used at high frequencies attention must be given also to the speed of operation of the discharge circuit because the discharge of C_2 by the trigger circuit must be carried out in a time that is small compared to $1/f$. In the case of the blocking oscillator when the critical voltage is reached as is determined by the cathode bias the tube becomes conductive and the circuit goes into violent oscillation which only lasts for one cycle because the grid is again driven to plate current cut-off. Since there is no grid resistor the tube remains blocked until the grid again receives a sufficiently high voltage from C_2 to offset the bias voltage. When the grid bias voltage becomes slightly more positive than the cut-off bias and plate current starts to flow, the grid voltage is raised still higher due to the regenerative character of the system and the plate current rapidly reaches saturation. The internal resistance of the trigger tube is low at positive grid voltage and the polarity is such that grid current flows from grid to cathode which permits a discharge of condenser C_2. When the tube reaches a saturated condition the plate current is unstable since no further plate current increase is possible and the grid voltage starts to fall. This fall is accelerated by the consequent falling plate current driving the grid to a value below the bias value where it remains until the C_2 voltage builds up again.

By taking the proper precautions it is possible to make the dividing factor of the frequency divider quite large, that is, something in the order of 60. We have found, however, that ordinarily a count of 8 or 16 is as high as it is usually desirable to go with one stage of division, especially since it is so easy to cascade divider circuits and thereby multiply the count or dividing number.

In the divider thus far described we have considered the input to the counter as being of rectangular wave form but no mention was made as to the ratio of the length of the flat top of the wave to the flat bottom. This is not important so a very sharp pulse wave such as is produced by the trigger circuit may be used to drive a similar divider circuit and make it readily possible to cascade dividers. It is advisable, however, to put a buffer tube between the trigger tube and the next counter.

We have found this divider circuit to be particularly useful in extending the range of frequency measuring instruments. For example, the Conn Chromatic Stroboscope is capable of measuring frequencies between 31.772 c.p.s. and 4066.8 c.p.s. with an accuracy within 1/20 of 1 percent. Sometimes it is necessary to measure the frequencies at say 30 kilocycles with the same accuracy and by means of the frequency divider we can extend the range of the stroboscope as far as we like. We have constructed other laboratory apparatus in which a synchronous motor is driven by a submultiple of the frequency of the input voltage to the instrument. The synchronous motor is capable of operating on frequencies between 20 and 65 c.p.s. so by means of adjustable frequency dividers the input frequency is divided by the proper number to make the input to the motor fall within its frequency range. The synchronous motor in turn drives a tachometer which accurately indicates its speed and the tachometer is a direct indication of the frequency of the signal applied to the instrument. We have used a sensitive magnetic drag tachometer as built by Kollsman and which is guaranteed to give an accuracy within $\frac{1}{4}$ of 1 percent of full scale value. We have also used a d.c. magneto as a tachometer and read the frequency on voltmeter scale. The d.c. voltage from the magneto is particularly valuable when we wish to

measure the rate of change of frequency because the d.c. voltage may be readily differentiated to obtain acceleration. The voltage applied to the synchronous motor in this instrument may drive an electric clock and in that way become a revolution counter which gives an extremely precise means of measuring frequency which is stable over long periods of time. One frequency measuring instrument used by us in our laboratory and making use of the frequency divider has an input frequency range from 20 c.p.s. to 33,280 c.p.s. and is extremely stable and accurate over long periods of time and with wide fluctuations of line voltage.

Improved Low Frequency Horn

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Hope, Arkansas

(Received September 10, 1942)

Last fall this writer described two designs for a low frequency horn adapted to operate in the corner of a room. The data presented consisted of experimental and computed performance on an experimental model and the computed performance of an improved design. The present paper describes the measured impedance of the new design and shows that this new design is a practical answer to the requirements of a speaker of this type. Minor discrepancies between the computed and measured performance are explained. Such discrepancies as exist are more of academic than practical interest. The improved design possesses all the characteristics which are desirable in a device of this class. This speaker, which is about the size of an ordinary radio console, has good response down to 40 cycles and an average efficiency from 40 to 400 cycles of about 30 percent, and a maximum variation of efficiency in this frequency range of only 2 decibels. Its normal environment, a room corner, utilizes room space which is ordinarily wasted. Observers have commented favorably upon both the artistic use of the space in the corner of the room and the acoustic performance of the instrument. With its high efficiency it is suitable for large scale sound reproduction.

INTRODUCTION

WHEN this writer described a low frequency horn adapted to operate in a room corner,[1] only a part of the experimental work had been completed. The data presented covered an experimental model. From the computed and measured response of this model the errors of the design were determined and an improved design worked out. Before construction and test of the improved design could be carried out, orders to active military duty were issued and it was felt best to report immediately as much data as had been accumulated. Subsequently it has been possible to construct and test a model of the improved design. This paper describes the results of the tests. Numerous but minor differences between measured and expected performance are pointed out and explained. For the design and construction elements of the improved speaker recourse should be had to the original paper.

MEASURED RESULTS

In Fig. 1 is shown the measured impedance of the improved horn. This may be compared with the computed impedance shown in Fig. 7 of the original paper. In Fig. 2 are shown the efficiencies computed from the measured impedance (curve 1) and computed from the design data (curve 2). Curve 2 is the same as that shown in Fig. 8 of the original paper. The curve of efficiency computed from the design constants shows an average efficiency of roughly 40 percent whereas the efficiency computed from the measured impedance shows efficiency of about 30 percent, or approximately $1\frac{1}{4}$ decibels less. The maximum variation in efficiency based on the measured impedance is only 2 decibels from about 40 to 400 cycles. This variation is too small for the human ear to detect even when listening to a

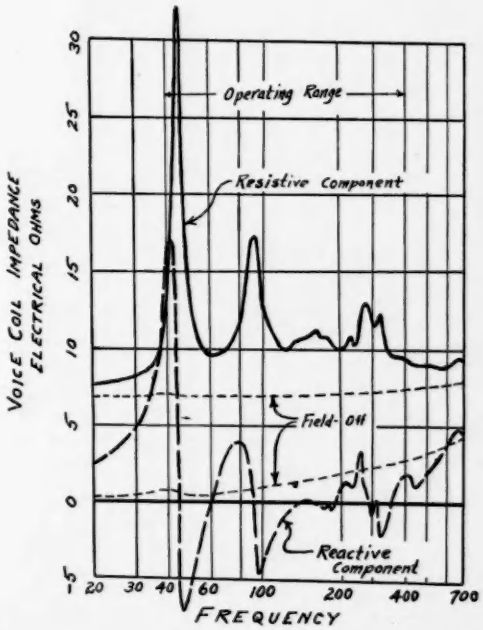

FIG. 1. Measured voice coil impedance of the driving unit operating in the improved horn. Solid curve, resistive component; dashed curve, reactive component; dotted curve, field off or blocked impedance curves. For computed impedance curves see Fig. 7 of the original paper.

[1] P. W. Klipsch, "Low frequency horn of small dimensions," J. Acous. Soc. Am. **13**, 137 (1941).

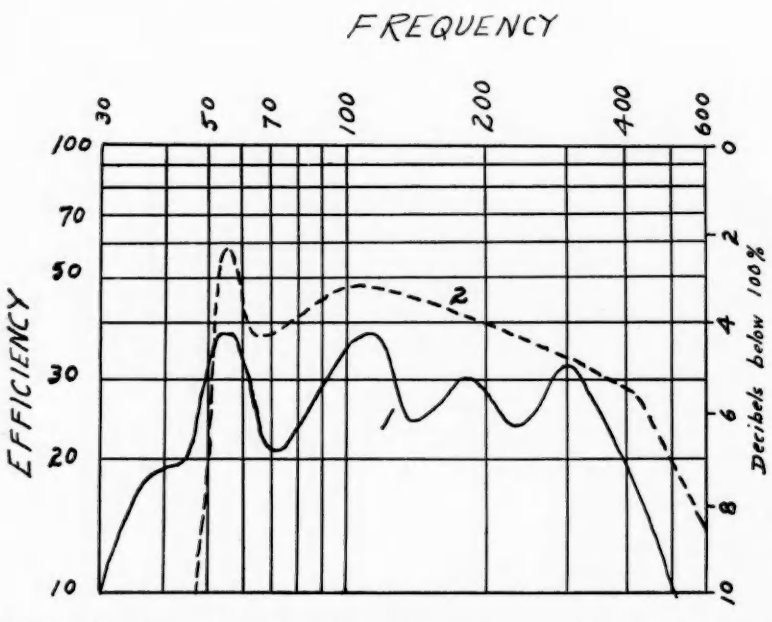

FIG. 2. Efficiency. Curve 1, absolute efficiency computed from the measured impedance using the Bostwick equation; curve 2, efficiency computed from the design data; voice coil resistance 6.5 ohms; power source impedance 6 ohms.

variable pitch tone produced by an oscillator and is of no consequence when listening to speech or music. Thus the improved horn is a very practical design.

This improved horn follows substantially the construction shown in Figs. 1 to 4 of the original paper. Some minor changes were necessitated due to the inability to obtain exactly the materials desired. For example, scrap $\frac{3}{4}$" plywood was used throughout, it being impossible to obtain thinner material either new or used. One minor change consisted of increasing the volume of the air chamber behind the cone by opening up the otherwise dead air spaces in the front of the cabinet lying on either side of the first horn section. This added approximately 600 cubic inches to the back air chamber. These openings must be sufficiently large so that they do not introduce an appreciable inertance between the two side air chambers and the back air chamber. A total area of opening of approximately 15 square inches is sufficient.

MULTIPLE TAPER

In the original paper it was implied that the initial taper was conical for the first foot or so. The purpose of this initial conical taper is to provide an effectively smaller throat and higher load impedance on the diaphragm at the higher frequencies whereby the efficiency at the higher frequencies is maintained at a high level. This follows the idea of Olson[2] who proposed the use of a short rapid taper close to the diaphragm. In the present case this initial taper consists of two parallel baffles and two straight diverging baffles so that the area increases linearly with the distance along the axis of the horn. This really corresponds to the "parabolic" rather than "conical" expansion which results in a smaller effective throat area for frequencies below 200 cycles and a larger effective throat area above 200 cycles than was supposed in the original paper. Thus the "rubber throat" is "stiffer" and the efficiency at the extreme ends of the frequency range can be expected to be somewhat lower than the computed values shown in Fig. 8 of the original paper. Probably from an academic standpoint an improvement in impedance can be made by making the first 12 to 14 inches of horn length in such form that the area increases as the square of the axial length instead of linearly. Practically however, such a change is not sug-

[2] H. F. Olson, "A horn consisting of manifold exponential sections," J. Soc. Mot. Pict. Eng. **30**, 551 (1938).

gested since the average efficiency from 40 to 400 cycles is around 30 percent and varies only 2 db within this frequency range.

EFFICIENCY

The efficiencies computed in Fig. 2 are based on an equation due to Bostwick.[3] This efficiency is given in Eq. (6) of my original paper. The value of efficiency so obtained is indicative of the total acoustical power output which can be produced from an electrical source with a given maximum undistorted electrical power output. Thus, for example, if the power amplifier has a normal power output of 10 electrical watts the acoustical power output would vary from 2 to 4 watts as the efficiency varies from 20 to 40 percent. Obviously the product of the indicated efficiency times the maximum electrical power output of the driving source is equal to the acoustical power output. This efficiency differs from the more generally used efficiency computed from $\mu=r_{EM}/(r_{EM}+r_0)$ where r_{EM} is the resistive component of the motional impedance and r_0 is the voice coil resistance with the diaphragm blocked. The latter ratio describes only the ratio of the acoustic power output to the electrical power input without considering the impedance variations which affect the ability of a device to absorb power from a source of a given impedance. The efficiency shown in Fig. 2 is really a performance characteristic.

DISCREPANCIES BETWEEN THEORY AND MEASUREMENT

When the original paper was written, it was expected that the additional taper in front of the "mouth," formed by the parallel side portions of the housing, would reduce the magnitude of the first peak in the throat impedance below the computed value so that the first trough in the electrical impedance would be raised. This is not evident from the measurements. It is believed that the explanation for this lies in the nature of the approximations used in deriving the expressions for the acoustic impedance at the throat of the horn. The same explanation gives the reason for the acoustic output below the nominal cut-off of the horn due to its taper. (See reference 10 of the original paper.) The throat impedances of infinite and finite horns are formulated and illustrated in various sources, the most complete being that of Olson.[4] All these data indicate that the radiation resistance drops to zero below the nominal cut-off due to taper rate of the horn. Yet it is well known that radiation occurs below this frequency. The improved horn has a cut-off due to flare of 50 cycles, but the impedance measurements and ear tests show that a strong fundamental is radiated down to 35 cycles. It must be concluded that the computed horn impedances are only qualitatively correct for frequencies within an octave of the low frequency cut-off. In view of this, one is impressed by the agreement between the measured electrical impedance shown in Fig. 1 and the computed impedance shown in Fig. 7 of the original paper. Probably the final rapid taper does contribute to a small degree in smoothing out the impedance curve but this small effect is obscured by other and larger differences between theory and measurement. It would be very instructive to see both the computed and measured acoustical and electrical impedances and the sound pressure data of a group of ideal horns such as described theoretically by Olson.[5] This and the original paper present two comparisons between computed and measured performance. In spite of the discrepancy it is possible, using the theoretical principles of acoustic elements, to design a horn whose measured performance is approximately predictable from the computed performance. The main differences lie in a slightly lower average efficiency than computed and the fact that the actual cut-off frequency is lower than the theoretical value.

DRIVING SOURCE IMPEDANCE

From the shape of the impedance curves and the nature of the equation for over-all efficiency, it is evident that this speaker should be driven by a low impedance amplifier. If beam or pentode tubes are used, sufficient negative feedback should be provided to reduce the output im-

[3] L. G. Bostwick, J. Acous. Soc. Am. 2, 243 (1930).

[4] H. F. Olson, *Elements of Acoustical Engineering* (Van Nostrand, 1940).

[5] H. F. Olson, RCA Review 1, No. 4, pp. 68 (1937), also reference 4, see above.

pedance to a value comparable to that of a triode stage. If the driving circuit impedance is high the response would be quite ragged. The design of a speaker for use with a high impedance driving circuit would necessitate a much larger structure since the requirement of a smoother impedance curve would necessitate a larger mouth and air column.

REMARKS

This horn was originally intended for home use. Its high efficiency makes it ideal likewise in applications requiring large scale reproduction of sound such as auditoriums of any size. As pointed out in the original paper this basic design may be adapted to the use of more than one diaphragm in each housing and several housings may be stacked or grouped for delivery of very large amounts of power and/or for wider angle of radiation. Such grouping of units will entail a smaller space requirement than that of the usual low frequency theater type speaker and thus should be a boon to many theater applications where stage space is at a premium.

This improved horn has been in use in the writer's living room for several months. It has elicited favorable comment both for its improved tone and for its eye appeal.

Applications and Limitations of Mechanical-Electrical Analogies, New and Old

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(Received October, 1942)

The general problem of mechanical-electrical analogies is discussed. The advantages of Firestone's mobility system over the older analogy is clearly demonstrated especially when used with the powerful methods of circuit analysis presented. Torsional and geared, as well as linear systems are discussed. It is shown that the choice of analogy to be used is usually one of convenience, but that certain systems intrinsically make only one analogy possible. It is further shown that the use of a mechanical-electrical analogy in a system involving both electromagnetic and electrostatic coupling is fundamentally impossible as the resulting circuit could not satisfy Maxwell's equations.

INTRODUCTION

IN the simple problems of mechanical vibrations due to impressed vibratory forces the usual method of solution is to set up the differential equations directly from Newton's fundamental laws of motion, and the solution, including evaluation of the constants from the boundary conditions, is very often accomplished by inspection. For more complex systems the equations of Lagrange will generally give the differential equations with little trouble (one of their chief advantages being the elimination of confusion in the sign of forces brought about via Newton's third law), but the solution of simultaneous differential equations in the number of coordinates necessary to describe a complex system presents a formidable problem, especially to the average engineer or physicist, unversed in the complexities of such equations.

A method which has come into rather extensive use in recent years for the solution of vibrations in linear mechanical systems is the mechanical-electrical analogy, wherein an electrical-circuit is set up as an analogy to the mechanical system, and the circuit is then solved by the rather powerful and distinctive operational tools available to the electrical engineer. Firestone[1] has listed these tools as:

"1. A set of conventionalized symbols with which the essential characteristics of a circuit can be set forth in the form of a circuit diagram.

"2. The concept of the potential difference across elements in the circuit as contrasted with the potential of points in the circuit relative to ground; the advantage here is that the relationship between the potential difference across an element and the current through it depends only on the characteristics of the element itself and not on the characteristics of the rest of the system.

"3. The use of complex numbers to represent simple harmonic voltages and currents, both the magnitude and phase of these quantities being represented by the absolute value and angle of the complex numbers.

"4. The concept of the "impedance" of an element, which is defined as the complex ratio of the voltage across the element to the current through the element; rules are developed for computing the impedance of series of parallel combinations of elements, and the current can then be found simply as the voltage divided by the impedance."

Unfortunately most of the more complex circuits resulting from the mechanical-electrical analogy are not readily solved by merely computing the impedance of simple series and parallel combinations, and many workers in this field are not aware that the electrical engineer also has at his command methods for directly evaluating the voltage or current at any point or line in any circuit due to any voltages or currents in any other part of the circuit. This paper attempts nothing more than to illustrate the application of these methods in the solution of linear mechanical vibrations. In particular, it is felt that Firestone's

* Written while in Research Laboratory, General Electric Company, Schenectady, New York.

[1] F. A. Firestone, J. App. Phys. 9, 373 (1938).

mobility method of computing the vibration of linear systems has failed to achieve the wider use it deserves because it utilizes the constant current generator, a device relatively unfamiliar to those outside of the rather specialized field of electrical networks.

MECHANICAL-ELECTRICAL ANALOGIES

In the early days of electricity the electrical entities of inductance and capacitance were often illustrated as analogous to the mechanical quantities of mass and compliance, electromotive force took its name from its similarity to a mechanical force, and a current of charges was often likened to a fluid flow of particles. The analogies were formally based on the similarity of the differential equation

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E \qquad (1)$$

for an inductance (L), a capacitance (C), and a resistance (R) in *series*, where Q is the charge flowing and E is the applied voltage, and the differential equation

$$M\frac{d^2x}{dt^2}+R\frac{dx}{dt}+\frac{1}{C}x=F=M\frac{dv}{dt}+Rv+\frac{1}{C}\int v\,dt \qquad (2)$$

for a mass (M), a linear spring (compliance C), and a dashpot (R) in *parallel* (Fig. 1), where x measures the displacement of the mass and F is the applied force.

Probably because of its senority this analogy has been called the "direct" relation between mechanical and electrical quantities;[2] however the term direct is somewhat of a misnomer since the velocity of a mechanical element relative to some reference point in a system corresponds to the current through an electrical element, so that the relative quantity of velocity is represented by the absolute quantity of current, whereas it might seem more logical to represent velocity by the voltage, which is also a relative quantity. The same line of reasoning shows that force is better represented by a current than a voltage. The so-called "inverse"[2] relation between mechanical and electrical quantities has been presented by both Hahnle[3] and Firestone,[1] Firestone designating it as the "mobility method" of computing vibrations. For reasons to be explained later, the old system will be called the "electrostatic" analogy, while the mobility system will be called the "electromagnetic" system.

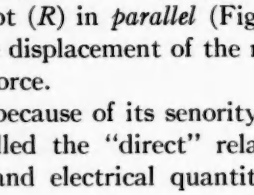


FIGURE 1.

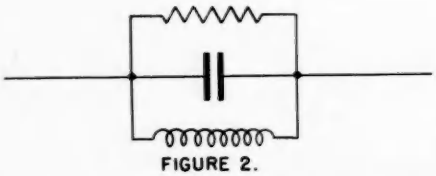


FIGURE 2.

FIG. 1. The elementary mechanical system having one degree of freedom and driven by a sinusoidal force.
FIG. 2. The elementary parallel electrical system.

The electromagnetic analogy has the distinct advantage of representing the *parallel* mechanical system of Fig. 1 as the *parallel* electrical circuit of Fig. 2, and its use prevents many of the errors which have baffled many of those attempting to apply the old mechanical-electrical analogy. (Nevertheless the old system is necessary for certain electrostatic and piezoelectric coupling elements.)[2,4]

The differential equation for Fig. 2,

$$C\frac{dE}{dt}+\frac{E}{R}+\frac{1}{L}\int E\,dt=I, \qquad (3)$$

is clearly similar to (2). The analogous quantities in both systems are given in Table I.

The fundamental imperfection of the old electrostatic analogy of velocity *across* mechanical elements being represented by current *through* electrical elements and force *through* mechanical elements being represented by voltage *across* electrical elements causes little trouble in simple systems; but it may also be said that the direct solution of these systems, sans analogies, gives even less trouble. In the case of a somewhat more complicated system, however, the newly initiated user of the old electrostatic analogy is very likely to become hopelessly confused and arrive at the

[2] W. P. Mason, *Electromechanical Transducers and Wave Filters* (D. Van Nostrand, New York).

[3] W. Hahnle, "Die Darstellung Elektromechanischer Gebilde Durch Rein Elektrische Shaltbilder," Wiss. Veroff. a.d. Siemens-Konzern (Julius Springer, Berlin, 1932), Vol. XI, No. I.

[4] *Electric Circuits*, M.I.T. Elec. Eng. Staff (John Wiley & Sons, Inc., New York, 1940).

TABLE I. Table of analogies.

Electrostatic analogy (old, "direct" analogy)	Mechanical system	Electromagnetic analogy ("mobility" analogy)
Voltage $=e$	Force $=f$	Current $=i$
$e=L\frac{di}{dt}$	$f=m\frac{dv}{dt}$	$i=C\frac{de}{dt}$
$e=Ri$	$f=R_M v$	$i=\frac{e}{R}$
$e=\frac{1}{C}\int idt$	$f=\frac{1}{K}\int vdt$	$i=\frac{1}{L}\int edt$
Kinetic energy $=\frac{1}{2}Li^2$	$\frac{1}{2}mv^2$	$\frac{1}{2}Ce^2$
Potential energy $\frac{1}{2C}[\int idt]^2$	$\frac{1}{2K}[\int vdt]^2$	$\frac{1}{2L}[\int edt]^2$
Current $=i$	Velocity $=v$	Voltage $=e$
Charge $=q=\int idt$	Displacement $=s=\int vdt$	Flux $=ie=\int edt$
Capacitance $=C$	Compliance $=K$	Inductance $=L$
Inductance $=L$	Mass $=m$	Capacitance $=C$
Resistance $=R$	Viscous resistance $=Rm$*	Conductance $=\frac{1}{R}$
Series impedance $=Z=R+j\left(\omega L-\frac{1}{\omega C}\right)$	Force/Velocity	Parallel admittance $=\frac{1}{R}+j\left(\omega C-\frac{1}{\omega L}\right)$

* This is only true when dealing with a purely resistive element, however our dampers are always assumed to be massless and inelastic.

most erroneous answers. In Fig. 3 is shown a fairly simple mechanical system; above it is the analogous electrical circuit via the old approach; and below is the circuit arrived at via the mobility approach. The marked advantages of the mobility approach are at once obvious. In spite of the niceties of the above pictorial analogy, a certain amount of judgment is necessary in drawing equivalent circuits. It should be noted that mechanical elements are in series when the same force acts through each of them, while the velocity across the combination is the *vector sum* of the velocities across the individual elements; elements are in parallel when the velocity across them is common, and the force through the combination is the *vector sum* of the forces through each of the elements. In particular, any mass has one terminal connected to ground, so that masses apparently in series as in Fig. 4 are schematically in parallel. These definitions follow directly from the fundamental analogy.

The use of either of the mechanical-electrical analogies is based primarily on the similarity of the differential equations of the respective system. Generally the choice is a matter of convenience and it is herein contended that the electromagnetic method is the more convenient. However the intrinsic elements of certain classes of problems make one and only one analogy applicable. This matter will be dealt with in the latter part of this paper.

APPLICATION OF THE MOBILITY METHOD

In applying the mechanical-electrical analogies it should be remembered that all springs are assumed to follow Hooke's law, and all mechanical resistance (friction or viscosity) is directly proportional to the velocity between the two surfaces or points under consideration; if both of these conditions are not satisfied Eq. (2), and consequently our analogy, is not valid. Almost all elastic elements follow Hooke's law under normal conditions, and most dampers such as oil, rubber, and felt have linear viscosities within a limited range.

All forces are assumed to be sinusoidal, so that all mechanical elements exhibit simple harmonic motion. If a force is not sinusoidal it must be represented by a Fourier series, or by the more powerful Fourier transform,[5] after which effects may be combined by superposition. Vibratory motion is then represented by the real part of the rotating vector $\epsilon^{j\omega t}$,[5] where $\omega=2\pi f$ is the angular frequency. The mobility of an element or combi-

[5] T. V. Kármán and M. A. Biot, *Mathematical Methods in Engineering* (McGraw-Hill, New York, 1940).

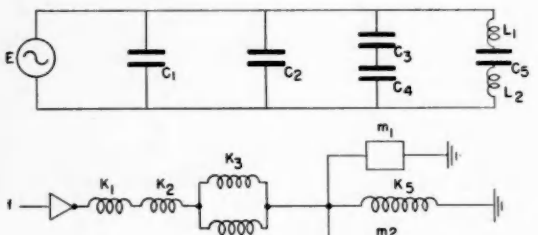


FIG. 3. A mechanical system (center) with its electrostatic analogy (above) and its electromagnetic analogy (below).

nation of elements is defined as the ratio of the velocity across an element to the force through the element. Firestone has called the real part of the mobility the responsiveness and the imaginary part the excitability.

CIRCUIT THEORY

The real advantages of the mechanical-electrical analogies, especially the electromagnetic method, only materialize in the case of rather complex mechanical systems which lead in turn to complex electrical circuits. Such circuits cannot be solved directly by Ohm's law, and it is necessary to apply Kirchhoff's laws, namely:

1. The *vector* sum of all the currents going into a point is equal to the current generated at that point.
2. The *vector* sum of all the voltage drops around any closed loop is equal to the voltage generated in the loop.

Firestone[1] states these laws as:

Force Law: The sum of all the forces acting on any junction point is zero.

Velocity Law: The sum of all the velocities across the structures included in any closed mechanical circuit is zero.

Unfortunately, the direct application of Kirchhoff's laws almost always leads to more currents than are necessary completely to solve the circuit, i.e., dependent variables are introduced. In the mechanical case this means there are more parameters than degrees of freedom. In the second example in this paper the direct application of Kirchhoff's laws leads to nine simultaneous equations, whereas the system can actually be solved via three independent variables.

Consider the circuit shown in Fig. 5. The impedances shown are complex, even though shown by resistance symbols. Straightforward application of Kirchhoff's laws leads to six equations since there are six currents corresponding to the six circuit elements shown, although it is unlikely that one would differentiate the current through Z_6 from that through Z_3. Two of the equations will be:

$$\begin{aligned} i_1-i_2-i_4&=0,\\ i_2-i_5-i_3&=0, \end{aligned} \tag{4}$$

where the minus signs designate currents leaving a junction. Clearly i_4 and i_5 can be represented as differences of circulating currents in the loops to which they are mutual. The remaining equations are obtained through Kirchhoff's second law:

$$\begin{aligned} i_1Z_1+(i_1-i_2)Z_4&=e_1+e_2,\\ (i_2-i_1)Z_4+i_2Z_2+(i_2-i_3)Z_5&=e_3-e_2,\\ (i_3-i_2)Z_5+i_3Z_3+i_3Z_6&=e_4-e_3, \end{aligned} \tag{5a}$$

where the polarity of the generators is such that the current would flow in phase with the voltages, in the directions shown, were the impedances pure resistances, a condition which is generally specified through the phase angle. Equation (5a) may be rewritten:

$$\begin{aligned} i_1(Z_1+Z_4)-i_2Z_4-i_3\cdot 0&=e_1+e_2,\\ -i_1Z_4+i_2(Z_4+Z_2+Z_5)-i_3Z_5&=e_3-e_2,\\ -i_10-i_2Z_5+i_3(Z_5+Z_3+Z_6)&=e_4-e_3, \end{aligned} \tag{6a}$$

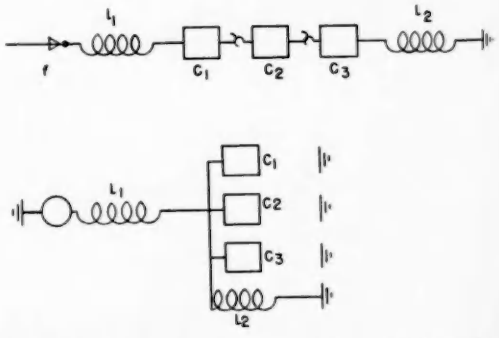


FIG. 4. Illustrating how masses apparently in series are actually in parallel, since the velocity of any mass is measured relative to ground.

or:

$$\begin{aligned} i_1Z_{11}-i_2Z_{12}-i_3Z_{13}&=E_1,\\ -i_1Z_{21}+i_2Z_{22}-i_3Z_{23}&=E_2,\\ -i_1Z_{31}-i_2Z_{32}+i_3Z_{33}&=E_3, \end{aligned} \tag{7a}$$

where Z_{11} represents the total impedance around loop 1, Z_{12} represents the impedance mutual to loops 1 and 2, and similarly for the other loops. For all linear circuit elements $Z_{12}\equiv Z_{21}$, etc., and we are interested only in linear elements. E_1, E_2, and E_3 represent the net voltages in their respective loops.

If we solve (7) by simple determinants we have:

$$\begin{aligned} i_1&=\frac{1}{\Delta}(E_1\Delta_{11}+E_2\Delta_{21}+E_3\Delta_{31}),\\ i_2&=\frac{1}{\Delta}(E_1\Delta_{12}+E_2\Delta_{22}+E_3\Delta_{32}),\\ i_3&=\frac{1}{\Delta}(E_1\Delta_{13}+E_2\Delta_{23}+E_3\Delta_{33}), \end{aligned} \tag{8a}$$

where:

$$\Delta=\begin{vmatrix} Z_{11} & -Z_{12} & -Z_{13}\\ -Z_{21} & Z_{22} & -Z_{23}\\ -Z_{31} & -Z_{32} & Z_{33} \end{vmatrix} \tag{9a}$$

and Δ_{kj} is $(-1)^{k+j}$ times the determinant obtained by eliminating the kth row and the jth column from Δ.

For the general case there will be n loops and n circulating currents, and:

$$i_k=\sum_{j=1}^{n}\frac{E_j\Delta_{jk}}{\Delta} \quad (k=1, 2, \cdots n), \tag{10a}$$

$$\Delta=\begin{vmatrix} Z_{11} & -Z_{12} & \cdots & -Z_{1n}\\ -Z_{21} & & & \cdot\\ \cdot & & & \cdot\\ \cdot & & & -Z_{(n+1)n}\\ -Z_{n1} & \cdots & -Z_{n(n-1)} & Z_{nn} \end{vmatrix}, \tag{11a}$$

so that we have available a method for writing down directly the current at any point in a circuit in terms of determinants and without resorting to the basic circuit equations.

While many engineers are familiar with the so-called "mesh-circuit" analysis culminating in (1a), few are aware that an exactly similar, but inverted, system of "junction analysis" leads to the solution of any circuit in terms of the voltages at certain junctions referred to ground and which is especially convenient when there are current generators in the circuit. Although any voltage generator in a circuit may be replaced by an equivalent current generator, the conversion involves mesh theory, and therefore a mesh solution is simpler. If a circuit has both voltage and current generators, inspection will usually show

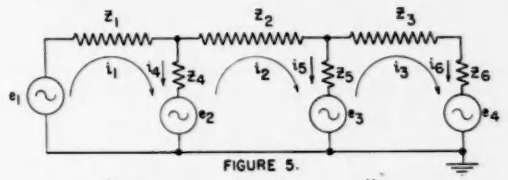


FIGURE 5.

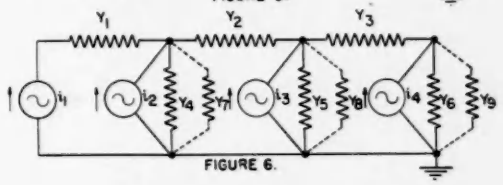


FIGURE 6.

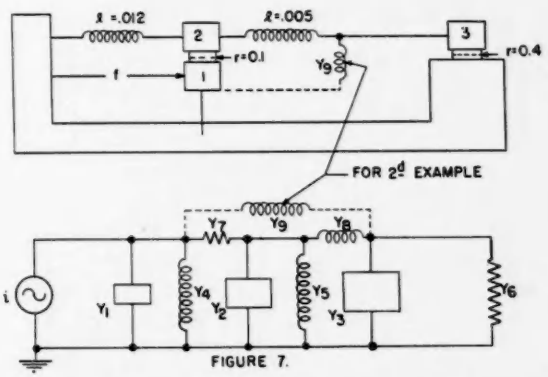


FIGURE 7.

FIG. 5. A three-mesh circuit which is best solved by (10a).

FIG. 6. A four-junction circuit which is best solved by (10b).

FIG. 7. Mechanical system and electromagnetic analogy. The addition of the spring Y_9 makes the solution by (10b) the only practical way.

which mode of attack is the simpler. Often a circuit will have fewer independent parameters when solved in terms of junction voltages and will lead to lower order determinants than would a mesh current solution.

In the application of the electromagnetic method the junction solutions are particularly advantageous not only because we are dealing with current generators, but also because the velocity (voltage) of all masses is referred to

ground, and because elements in parallel (common voltage) are more common than elements in series (common current). Accordingly the electromagnetic method of computing vibrations is not only easier to set up, and therefore less prone to error, than the old electrostatic method, but also almost always leads to lower order determinants in the solution and is therefore quicker and less prone to error on a second count.

Consider the circuit of Fig. 6, which is similar to Fig. 5 except we now have current generators (a current is always injected at a junction) and admittances rather than impedances are specified. The solution will be made in terms of the voltages (with respect to ground) at junctions *1*, *2*, *3*, and *4*. Since the circuit chosen for iilustration has more junctions than meshes it is necessary to use four voltages in the solution where only three currents were necessary. However if we place impedances from points *2*, *3*, and *4* to ground the circuit would still require only four voltages, whereas six currents would be necessary; such cases are prevalent in mechanical systems.

Ohm's law in admittance terms is:

$$i=eY. \tag{12}$$

Accordingly Kirchhoff's first law gives for Fig. 7:

$$\begin{aligned}(e_1-e_2)Y_1&=i_1,\\ (e_2-e_1)Y_1+e_2Y_4+(e_2-e_3)Y_2&=i_2,\\ (e_3-e_2)Y_2+e_3Y_5+(e_3-e_4)Y_3&=i_3,\\ (e_4-e_3)Y_3+e_4Y_6&=i_4.\end{aligned} \tag{5b}$$

Equation (5b) may be rewritten:

$$\begin{aligned}e_1Y_1-e_2Y_1-e_3\cdot 0-e_4\cdot 0&=i_1,\\ -e_1Y_1+e_2(Y_1+Y_4+Y_2)-e_3Y_2-e_4\cdot 0&=i_2,\\ -e_1\cdot 0-e_2Y_2+e_3(Y_2+Y_5+Y_3)-e_4Y_3&=i_3,\\ -e_1\cdot 0-e_2\cdot 0-e_3Y_3+e_4(Y_3+Y_6)&=i_4,\end{aligned} \tag{6b}$$

or:

$$\begin{aligned}e_1Y_{11}-e_2Y_{12}-e_3Y_{13}-e_4Y_{14}&=I_1,\\ -e_1Y_{21}+e_2Y_{22}-e_3Y_{23}-e_4Y_{24}&=I_2,\\ -e_1Y_{31}-e_2Y_{32}+e_3Y_{33}-e_4Y_{34}&=I_3,\\ -e_1Y_{41}-e_2Y_{42}-e_3Y_{43}-e_4Y_{44}&=I_4,\end{aligned} \tag{7b}$$

where Y_{11} is the total admittance at the junction *1*, Y_{12} is the mutual admittance to junctions *1* and *2*, etc., as before, $Y_{12}\equiv Y_{21}$, etc. Notice the inverse relation of (6a) and (7a) to (6b) and (7b).

$$e_1=\frac{1}{\Delta}(I_1\Delta_{11}+I_2\Delta_{21}+I_3\Delta_{31}+I_4\Delta_{41}),\ \text{etc.,} \tag{8b}$$

where now:

$$\Delta=\begin{vmatrix}+Y_{11}-Y_{12}-Y_{13}-Y_{14}\\ -Y_{21}+Y_{22}-Y_{23}-Y_{24}\\ -Y_{31}-Y_{32}+Y_{33}-Y_{34}\\ -Y_{41}-Y_{42}-Y_{43}+Y_{44}\end{vmatrix} \tag{9b}$$

and Δ_{kj} is as before. For a circuit having n junctions we have:

$$e_j=\sum_{k=1}^{n}\frac{I_k\Delta_{kj}}{\Delta}\quad(j=1,2,\cdots n), \tag{10b}$$

which is the inverse of (10b) where now:

$$\Delta=\begin{vmatrix}Y_{11} & -Y_{12} & \cdots\\ -Y_{21} & \cdot & \cdot\\ \cdot & \cdot & \cdot\\ \cdot & \cdots & Y_{nn}\end{vmatrix}. \tag{11b}$$

In applying the electromagnetic method there will usually be only one applied force, so that (10b) will have only one term different from zero. For the case of more than one force in a system the terms of (10b) represent the respective effects of each force, i.e., the principle of superposition is clearly demonstrated.

Not only does the mobility method often lead to lower order determinants, but, in addition, the individual terms in the determinant are more easily computed since mechanical elements in parallel (but not requiring an extra mesh) must have their mobilities calculated from the parallel impedance formula:

$$\frac{1}{Z}=\frac{1}{Z_1}+\frac{1}{Z_2}+\cdots+\frac{1}{Z_n}, \tag{13}$$

whereas as parallel admittances add directly:

$$Y=Y_1+Y_2+Y_3+\cdots+Y_n. \tag{14}$$

If, however, admittances occur in series:

$$\frac{1}{Y}=\frac{1}{Y_1}+\frac{1}{Y_2}+\frac{1}{Y_3}+\cdots+\frac{1}{Y_n}. \tag{15}$$

Fortunately, as already stated, mechanical elements usually occur in parallel.

EXAMPLES

The first step in solving a vibration problem is to set up the equivalent circuit, being careful to refer all masses to ground. Generally, most of the elements in the system, including the exciting force will have one terminal connected to ground so that the circuit is best drawn with one long, common line; this also serves to make the junctions clear. For each junction of mechanical elements, other than the common or ground, we will have a variable, namely, the velocity of that point. The velocities of the junctions are then obtained via (10b); generally each junction will be the terminal of a mass, so that we have immediately the velocities of all masses. To find the force through any element we merely apply (12) where e is the relative velocity between the two points.

For purposes of comparison we choose first an example solved by Firestone[1] shown in Fig. 7. A one-pound mass is supported by a leaf spring with a compliance of 0.054 in./lb. The motion of this first mass is transmitted to the second mass of 2 lb. through a viscous film having a "responsiveness" of 0.1 in./sec./lb., and this second mass is connected to the frame through a spring of 0.012 in./lb. and to a third mass of 3 lb. through a spring of 0.005 in./lb., this third mass being supported by the machine frame through a viscous film of 0.4 in./sec./lb. A sinusoidal force having a frequency of 15.91 cycles/sec. and an amplitude of 10 lb. acts on the 1 one-pound mass, reacting against the frame.

The angular frequency $\omega=2\pi 15.91=100$, and the admittances of the elements are tabulated below. The admittance of a spring is $-1/j\omega l$, where l is the compliance, or if we use the spring constant $p(p=1/l)$ the admittance of a spring is $-j(p/\omega)$, the admittance of a mass is $j\omega l=j\omega(w/g)(g=386$ in./sec.$^2)$, and the admittance of a resistor is simply $1/r$.

$$Y_1=j100\times\frac{1}{386}\times 1=j0.26,$$

$$Y_2=j100\times\frac{1}{386}\times 2=j0.52,$$

$$Y_3=j100\times\frac{1}{386}\times 3=j0.78,$$

$$Y_4=-j\frac{1}{100\times 0.054}=-j0.185,$$

$$Y_5=-j\frac{1}{100\times 0.012}=-j0.835,$$

$$Y_8=-j\frac{1}{100\times 0.005}=-j2.0,$$

$$Y_7=\frac{1}{0.1}=10,$$

$$Y_6=\frac{1}{0.4}=2.5.$$

Next we obtain the elements of Δ, where our three junctions are the terminals of the three masses:

$$Y_{11}=Y_1+Y_4+Y_7=10+j0.07,$$

$$Y_{22}=Y_2+Y_5+Y_7+Y_8=10-j2.31,$$

$$Y_{33}=Y_3+Y_6+Y_8=2.5-j1.22,$$

$$Y_{12}=Y_{21}=Y_7=10,$$

$$Y_{23}=Y_{32}=Y_8=-j2.0,$$

$$Y_{13}=Y_{31}=0,$$

$$\Delta=13.3-j56.2=57.8\angle -76.7°.$$

We now apply (10b). (Note that it is not necessary to find e_1 and e_2 before finding e_3.)

$$e_1=\frac{10\epsilon^{j100t}(-1)^{1+1}}{57.8\angle -76.7°}\begin{vmatrix}(10-j2.31) & j2\\ j2 & (2.5-j1.22)\end{vmatrix}=5.5\angle 42° \text{ in./sec.}$$

$$e_2=\frac{10\epsilon^{j100t}(-1)^{1+2}}{57.8\angle -76.7°}\begin{vmatrix}-10 & j2\\ 0 & (2.5-j1.22)\end{vmatrix}=4.8\angle 51° \text{ in./sec.}$$

$$e_3=\frac{10\epsilon^{j100t}(-1)^{1+3}}{57.8\angle -76.7°}\begin{vmatrix}-10 & (10-j2.31)\\ 0 & j2\end{vmatrix}=3.46\angle -13° \text{ in./sec.}$$

The operator ϵ^{j100t} merely implies simple harmonic motion, so that our answers are merely amplitudes of the respective velocities with phase displacements from the exciting force.

By applying (12) we obtain the force acting through the film Y_7:

$$i_7=(e_1-e_2)\,Y_7=10.3\angle -1.7^\circ \text{ lb.}$$

The force through the spring Y_8:

$$i_8=(e_2-e_3)\,Y_8=9.1\angle 4.2^\circ \text{ lb.}$$

The force through the film Y_6:

$$i_6=e_3 Y_6=8.65\angle -103^\circ \text{ lb.}$$

The displacement amplitudes are found by dividing by the operator $j\omega$ since this operator denotes differentiation with respect to time. Another way of seeing this is to remember that the velocity amplitude in simple harmonic motion is ω times the displacement amplitude and leads it by ninety degrees. We then have:

$$s=V/j\omega=e/j\omega. \tag{16}$$

Since voltage is equal to the rate of change of flux, we see that the electrical analogy of displacement in the mobility method is flux.

The displacement amplitudes are found to be:

$$s_1=0.055\angle -47.6^\circ \text{ inches,}$$

$$s_2=0.048\angle -39^\circ \text{ inches,}$$

$$s_3=0.035\angle -103^\circ \text{ inches.}$$

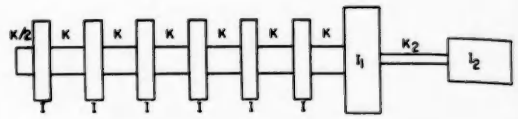
The average power supplied to a circuit or to any element in a circuit may be computed by any of three formulae:

$$p=\tfrac{1}{2}|i|^2 r, \tag{17a}$$

$$p=\tfrac{1}{2}(ei^*) \text{ real}, \tag{17b}$$

$$p=\tfrac{1}{2}ei\cos(\theta e-\theta i), \tag{17c}$$

where i^* denotes the conjugate of i, obtained by changing the sign of the imaginary part. Equation (17a) is used in order to find the power dissipated in an element or in a network, the real part of whose impedance is known, while (17c) or (17b) can be used to find the power dissipated in any two-terminal network when the voltage and current are known. The $\frac{1}{2}$ factors are due to



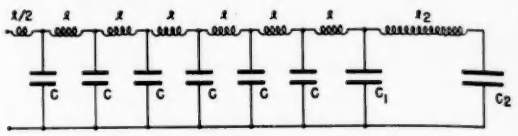
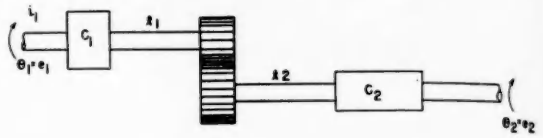


FIGURE 8.



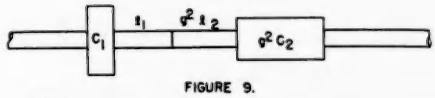


FIGURE 9.

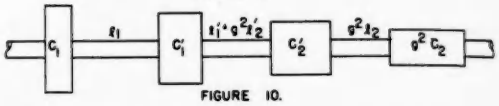


FIGURE 10.

FIG. 8. A six-cylinder engine with flywheel and propeller load and its analogy.
FIG. 9. A geared system and its equivalent.
FIG. 10. The system of Fig. 9 when the gears have mass and elasticity.

the fact that our quantities are peak and not R. M. S.

The average power delivered by the source is best found via (17c):

$$p_{\text{source}}=\tfrac{1}{2}e_1 i\cos\theta e_1$$

$$=\tfrac{1}{2}\times 5.5\times 10\cos 42^\circ=20.4 \text{ lb. in./sec.}$$

The power dissipated in Y_7 is:

$$p_7=\tfrac{1}{2}|10.3|^2\times 0.1=5.3 \text{ lb. in./sec.}$$

The power dissipated in Y_6 is:

$$p_6=\tfrac{1}{2}|8.65|^2\times 0.4=15.0 \text{ lb. in./sec.}$$

Since the films are the only dissipative elements in the system the total of p_6 and p_7 should be p_{source} which it is, allowing for reasonable errors in calculations.

In the example just given the solution could also have been obtained by finding the impedance offered by the system to the source and then working back, although the calculation of this impedance requires troublesome computations due to the various series and parallel combina-

tions involved. However if we add the spring shown dotted in Fig. 7 any such straightforward calculation becomes impossible, whereas the junction solution requires only that certain of the terms in the determinant be changed:

$$Y'_{11}=Y_{11}+Y_9, \quad Y'_{12}=Y_{12},$$
$$Y'_{22}=Y_{22}, \quad Y'_{23}=Y_{23},$$
$$Y'_{33}=Y_{33}+Y_9, \quad Y'_{13}=Y_{13}+Y_9=Y_9.$$

The solution can then be carried out exactly as before.

The electromagnetic method is just as convenient with torsional as with linear vibrations. The same symbols are used, analogies now being torsional: e is the angular velocity across an element in radians per second; i is the torque through an element, usually in pound-inches; Z is the torsional mobility in rad./sec./lb. in.; Y is the torsional admittance lb. in./rad./sec.; c is the moment of inertia in slug-inches ($=1/g$ lb. in.^{2}); l is the torsional compliance in rad./lb. in.; and r is the torsional resistance or responsiveness in rad./sec./lb. in.

Consider the system of Fig. 8 which represents a six-cylinder Diesel engine with a flywheel directly coupled to a ship propeller through a long shaft. This same problem is treated elsewhere, by the method of Holzer[6] and by the method of matrices.[7] The first six disks (I) are the effective representations of the piston, crank, and connecting rod of each cylinder, the large disk (I) is the flywheel, and I_2 is the propeller. The engine shaft has a compliance of K, and the propeller shaft has a compliance K_2. The equivalent circuit is shown below the mechanical system. The resulting determinant is of the eighth order, however, 42 of its 64 terms are zero, 10 are equal to $-1/\omega l$, 2 to $-2/\omega l$, 1 to $+2/\omega l$, 4 to $(\omega c-2/\omega l)$, 1 to $(\omega c-3/\omega l)$, 2 to $-1/\omega l_2$, 1 to $(\omega c_1-1/\omega l-1/\omega l_2)$, and 1 to $(\omega c_2-1/\omega l_2)$; accordingly calculation is considerably simplified.

From (10b) it is seen that the velocity of all of the disks is infinite for any arbitrary torque, provided that $\Delta=0$, i.e., the vanishing of Δ is the criterion of resonance. The methods given in the above references greatly facilitate the solution of the transcendental equation represented by Δ. Reference 1 treats the case sans flywheel and propeller.

Very often a mechanical system involves gears, as in Fig. 9. If the gear ratio from shaft l_1 to l_2 is g, then the torque i_2 is i_1/g and the displacement e_2 is ge_1; then we see that the torque transferred through the gears is

$$i_1=gi_2=ge_2Y_2=g^2e_1Y_2$$

and

$$i_1/e_1=Y_1=g^2Y_2. \tag{18}$$

Accordingly, the gears are removed from the system by multiplying the admittances beyond by g^2. The equivalent circuit is shown in Fig. 9. If an element has two sets of gears between it and the exciting force its admittance is multiplied by $g_1^2g_2^2$. If there is more than one source a reference source is chosen arbitrarily, forces separated from the reference source by gears then have their amplitudes divided by g, and velocity generators (discussed further below) have their amplitudes multiplied by g. If the gears are considered to have mass and compliance (l_1', c_1', l_2', c_2') the equivalent circuit is modified as shown in Fig. 10.

Occasionally a mechanical system will have a velocity source for excitation, as is the case of any object elastically coupled to a relatively large vibrating mass. For such a system it is then advisable to use (10a). If both velocity and force excitation are present in a system (although this may be tantamount to irresistable force meeting immovable object) it will be necessary, in most cases, to use Thevenin's theorem[1,2] to convert to all force or all velocity sources, a relatively complicated process. Firestone[1] has stated Thevenin's theorem, along with the reciprocity, compensation, and superposition theorems, for mechanical systems. He has also discussed mechanical impedance matching. In addition any of the methods and theorems of network analysis[2,4,8] are applicable to mechanical systems.

One of the great advantages of the mechanical-electrical analogy lies in the ease of making electrical measurements, and a complicated mechanical or electro-mechanical system that might present a solution of very high order on paper is

[6] J. P. Den Hartog, *Mechanical Vibrations* (McGraw-Hill, New York, 1934), p. 227.

[7] L. A. Pipes, "Matrix theory of torsional oscillations," J. App. Phys. **13**, 434 (1942).

[8] E. A. Guillemin, *Communication Networks*, Vol. II (John Wiley & Sons, Inc., New York, 1935).

easily solved on the so-called "*A-C* Board" or network analyzer.

LIMITATIONS OF ANALOGIES

It has already been pointed out that the mechanical-electrical analogies are subject to certain definite restrictions such as linear springs and linear damping, but in the examples presented so far we have had our choice between the two analogies, since either analogy led to completely similar differential equations. Suppose now that we are dealing with a system involving electromagnetic coupling. The fundamental equations are:[4]

$$e = Blv, \tag{19}$$

$$f = Bli, \tag{20}$$

where a conductor of projected length l moves with a velocity v in a magnetic flux of density B. It is immediately clear that electromagnetic analogy is the only one suitable for treating systems involving electromagnetic coupling.

In the case of electrostatic coupling, as in a condenser microphone, the displacement current is equal to the rate of change of the coupling field and is therefore proportional to the velocity. Accordingly we are forced to use the old or "direct" mechanical analogy.

Although Mason[1] points out that the above analogies are advisable in the respective cases, he does not state that these choices are imperative, a statement which is clearly made in reference 4.

Since electromechanical conversion based on electromagnetic coupling is far more important from an engineering point of view both from the standpoints of number of applications and capital involved we have one more powerful argument for the electromagnetic (mobility) method of computation. References 2 and 4 give numerous examples of electromagnetic (including magnetostrictive) and electrostatic (including piezoelectric) coupling.

Due to the above limitations it would seem reasonable to call the old or "direct" analogy the electrostatic-mechanical analogy and the mobility analogy the electromagnetic-mechanical analogy. These names are hereby proposed.

In the case of fluid mechanics (including acoustics) we have pressure instead of force and volumetric flow instead of velocity. Since probably neither electromagnetic or electrostatic coupling will exist the choice of analogy is one of convenience, although either type of coupling may occur in acoustical problems.

The result of the above limitations makes it fundamentally impossible to apply an electrical analogy to a mechanical system having both electrostatic and electromechanical coupling since such an analogy could not be made to satisfy Maxwell's equation

$$\nabla \times (\bar{B}/\mu) - (\partial/\partial t)(\epsilon \bar{E}) = \bar{J}, \tag{21}$$

which is a necessary condition for all electromagnetic circuit theory.[9] (In (21) μ is the permeability, ϵ is the dielectric constant, $\bar{J}$ is the current density, $\bar{E}$ is the electric flux, and $\bar{B}$ is the magnetic flux.) Fortunately practical problems of this nature never arise (although the swaying of a transmission line might be so considered), as electromagnetic forces are seldom found simultaneously with electrostatic forces of the same magnitude. That a fundamental limitation should be placed on our analogies should not be surprising when it is remembered that the analogies were based purely on similarity of the differential equations.

CONCLUSIONS

The advantages of the electromagnetic (mobility) method over the older mechanical-electrical analogy have been clearly demonstrated, particularly in connection with the junction method of circuit solution. It has been shown that the mobility method is imperative in cases involving electromagnetic coupling and that the old electrostatic analogy is necessary when electrostatic coupling is present. When both of these types of coupling occur simultaneously it is fundamentally impossible to use a mechanical-electrical analogy since the resulting circuit could not satisfy Maxwell's equations. Accordingly it is concluded that the present mechanical-electrical analogies are not of a basic and fundamental nature.

[9] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).

Acoustical Society News

Spring Meeting

The next meeting of the Acoustical Society is planned for an early date in May. It is hoped that it will be possible to hold the meeting in New York City but this is uncertain not because of the inability of the executive council to make up its mind but because of a request that final decision be deferred until the transportation situation along the Atlantic Seaboard becomes more clarified.

The two committees in charge of this meeting are:

Papers Committee

H. BURRIS-MEYER
G. NIXON
H. F. OLSON, *Chairman*
P. E. SABINE

Arrangements Committee

LONSDALE GREEN, JR., *Chairman*
R. L. CARDINELL

Any inquiries or suggestions regarding the meeting may be addressed to the chairman of these committees.

E. C. WENTE

Noise Abatement Resolution

At the meeting of the Executive Council of the Acoustical Society on October 24, 1941, Dr. Paul E. Sabine, as the Society's representative on the Advisory Committee of the Noise Abatement Council, reported on certain requests made of this Society by the Noise Abatement Council. The Executive Council chose to appoint a committee to investigate these requests further. Dr. Sabine was appointed Chairman of this Committee and chose six other members. The Committee met in Ann Arbor on May 15, 1942 and reported to the Executive Council. As a result of this report, the three following resolutions were adopted:

RESOLVED, That the Acoustical Society of America endorses the object of the National Noise Abatement Council as expressed in Article II of their By-Laws, as follows:

The object of the National Noise Abatement Council shall be to promote a national consciousness of the need for noise abatement and control; to publicize the causes and costs of noise and its evil effects on individual health and efficiency; to publicize and disseminate information to the public concerning the advantages and benefits of quiet, and ways and means for controlling and abating objectionable noise on city streets and in public places, in factories, offices, schools, and homes. It will not be the policy of the National Noise Abatement Council to promote the interests of any individual member or group of members, but rather the scope of its activity will encompass all interests and all phases of the problem of noise abatement and control.

RESOLVED, That there is a need in connection with noise abatement and other activities in which the Society is interested for the establishment of recommended maximum permissible noise levels for noise in rooms of different types of occupancy, transportation vehicles, etc., and that the Society instructs the Secretary to request the American Standards Association to establish a committee to formulate standards of this type.

RESOLVED, That in view of the undesirable effect of unnecessary noise on public health and on civilian morale, and of the further fact that the difficulty of air raid signaling is increased by excessive noise, the Acoustical Society of America hereby endorses the movement for noise abatement as a civilian defense measure, and recommends to the Office of Civilian Defense that appropriate steps be taken through proper channels to promote the abatement of unnecessary noise.

Whereas the viewpoint of some is that the reduction of noise is a luxury which, in view of our war effort, we cannot afford, the fact that the area of coverage of air raid warning devices is considerably dependent on the level of city noise gives the abatement of noise a practical advantage inherent to war time.

Incentive for Writing Papers on Acoustical Subjects

The Editor has noted the following closing paragraph in a letter from an author whose papers have been referred to in this Journal several times in the past year or two:

"I have rather been bombarding you lately with papers, but I think you may take it that they have now pretty well come to an end. I have found the writing of them a considerable recreation under the strain of war work. They have occupied the evenings when one is confined to the house by the black-out."

Yours sincerely,
L. S. LLOYD
Gerrards Cross, Bucks, England.

New Members

Melville Berman
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Radio Engineering Student
RCA Institute
76 Varick Street
New York, New York

Herbert H. Blossom
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Pittsburgh, Pennsylvania
Acoustical Engineer
Union Acoustical Company

Charles Cyril Curran
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District Manager
Dominion Sound Equipments, Ltd.

Julio del Río
Calle Callao 397 (Providencia)
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Engineer, Casilla 1407

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Research Physicist

Gregory L. Irsky
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Chairman, Motion Picture Group
Soviet Government Purchasing Commission

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Luther C. Smith, Jr.
Stromberg-Carlson Tel. Manufacturing Company
Rochester, N.W.
Radio Engineer

Michael Z. Wysotzky
3355—16th Street, N.W.
Washington, D.C.
Electrical and Sound Motion Picture Engineers
Soviet Government Purchasing Commission

Resigned as of 4–30–42: Charles W. McWilliams (Member)
Deceased: S. P. Walker (Member)

Fellows	166
Members	690
Total membership	856

Current Publications on Acoustics

References to Contemporary Papers on Acoustics

2. Architectural Acoustics

2.2 **Contribution to the Acoustics of Radio Studios: III —Reverberation Theories and Reverberation Investigations.** W. Furrer. Schweizer Arch. f. Angew. Wiss. u. Tech. **8**, 99–109 (April, 1942); Wireless Engineer **19**, Abs. 3026 (1942).

2.2 **The Acoustics of Large Buildings.** K. W. Wagner. Génie Civil **119**, No. 5/6, 66 (Jan. 31/Feb. 7, 1942), Wireless Engineer **19**, Abs. 3027 (1942).

2.3 **Acoustic Properties of the Gol Gumbaz, Bijapur (built as the tomb of a sultan in 1656).** G. R. Paranjpe. J. Univ. Bombay **10**, 124–132 (Nov. 1941). Sci. Abs. **A45**, 2061 (1942).

3. Books and Bibliographies

3.1 **Review: Practical Acoustics and Planning Against Noise, by Hope Bagenal.** N. Fleming. Nature **150**, 193 (Aug. 15, 1942).

3.1 **Text-Books of Acoustics** (Review of *Acoustics*, by Alexander Wood and *A Text-book of Sound*, by A. B. Wood). W. H. George. Nature **150**, 248 (Aug. 29, 1942).

3.1 **Book Review: Fundamentals of Vibration Study.** R. G. Manley. Nature **150**, 392 (Oct. 3, 1942).

4. Ear and Hearing

4.1 **Errors in Perceiving the Temporal Orders of the Auditory and Visual Stimuli.** L. Bald et al. J. Ap. Psychol. 26, 382–388 (June, 1942).

4.1 **Interference and Distortion in the Cochlear Responses of the Pigeon.** C. W. Bray and W. R. Thurlow. J. Comp. Psychol. **33**, 279–289 (April, 1942).

4.1 **Temporary Deafness in Birds during Mating Song; Measuring the Effect of Opening the Mouth of the Pigeon on the Sensitivity of the Ear of the Bird to Sound.** C. W. Bray and W. R. Thurlow. Aukens **59**, 379–387 (July, 1942).

4.1 **The Influence of Ultra-Sounds on the Ear of Animals.** K. A. Dernnova and A. I. Titov. Zeits. usn. Bol. **16**, 104–113 (1939).

4.1 **On the Dynamics of Induction Relations in the Cerebral Cortex Involved in the Phenomenon of Acoustic Illusion of Rhythm Frequency.** I. L. Korotkin. Fiziol. Zh. U.S.S.R. **28**, No. 5, 421–430 (1940).

4.1 **Physiological and Motor Responses to a Regularly Recurring Sound: A Study in Monotony.** G. D. Lovell and J. J. B. Morgan. J. Exp. Psychol. **30**, 435–451 (June, 1942).

4.1 **An Alternate Approach to the Mathematical Biophysics of Perception of Combinations of Musical Tones.** N. Rashevsky. Bull. math. Biophys. **4**, 89–90 (1942).

4.1 **Auditory Rivalry.** S. Sato. Z. Otol. Tokyo **46**, 79–80 (1940).

4.2 **The Response of Single Auditory Nerve Fibers to Acoustic Stimulation.** R. Galambos and H. Davis. Fed. Proc. Am. Soc. exp. Biol. **1**, No. 1, Part II, 27–28. Abs. (1942).

4.2 **Observations on the Acoustic Movements of the Human Sound Conducting Apparatus.** H. G. Kobrak. Ann. Otol. etc. St. Louis, **51**, 554–555 (1942).

4.2 **Frequency-Localization in the Auditory Cortex of the Monkey.** J. C. R. Licklider and K. D. Kryter. Fed. Proc. Am. Soc. exp. Biol. **1**, No. 1, Part II, 51. Abs. (1942).

4.2 **Cancellation of the Electrical Cochlear Response of Simultaneous Stimulation with Air- and Bone-Conducted Sound.** K. Lowy. Fed. Proc. Am. Soc. exp. Biol. **1**, No. 1, Part II, 51–52. Abs. (1942).

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4.2 **Stapedius Muscle in Relation to Sound Conduction.** E. G. Wever and C. W. Bray. J. Exp. Psychol. **31**, 35–42 (July, 1942).

4.3 **Significance of Audible Onset as a Cue for Sound Localization.** D. P. Boder and I. L. Goldman. J. Exp. Psychol. **30**, 262–272 (March, 1942).

4.3 **Distance Determination by Means of Sound.** D. Elkin and R. Tagamlitzkaya. Z. usn. Bol. **16**, 195–200 (1939).

4.4 Many articles on **Deafness and Hearing Aids.** Volta Review. Monthly.

4.4 **The Effect of Flight upon Hearing.** P. A. Campbell. J. Aviat. Med. **13**, 56–61 (1942).

4.4 **Re-Education of Defective Hearing.** L. D. Goodfellow. J. Psychol. **14**, 53–58 (July, 1942).

4.4 **Studies in the Psychology of the Deaf.** F. Heider and G. M. Heiger. Psychol. Monogr. **53**, No. 5, 158 (1941).

4.4 **A New Telephone Set for the Hard of Hearing.** Alfred Herckmans. Bell Lab. Rec. **21**, No. 2, 45–48 (Oct. 1942).

4.4 **Studies on Pupils in the Pennsylvania School for the Deaf. IV. Mechanism of Inheritance of Deafness.** W. Hughson, A. Ciocco, and C. E. Palmer. Arch. Otolaryng. Chicago **35**, 871–882 (1942).

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5.1 **New Principles of Construction of Studio Electro-acoustical Installations.** F. de Fremery and J. W. G. Wenke. Alta Frequenza **10**, 751–752 (Dec. 1941).

5.1 **Über die Bestimmung Akustischer Resonanzen nach der Hitzdrahtmethode.** A. Loebenstein. Helv. Phys. Acta **15**, 321–322 (1942).

5.8 **Two or More Loud Speakers at the Amplifier Output.** E. de Gruyter. Bull. Assoc. Suisse des Elec. **33**, 103–107 (Feb. 25, 1942); Wireless Engineer **19**, Abs. 2705 (1942).

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5.8 **Radio Bibliography. Part 6—Filters, Sound, Loud-speakers.** F. X. Rettenmeyer. Radio No. 273, 25–26, 39–40 (Oct. 1942).

5.9 **Making a Moving-Coil Pick-Up.** J. Brierly. Wireless World **48**, 154–156 (July, 1942).

5.10 **Acoustic Aircraft Detection.** QST **26**, 44, 108–110 (Oct. 1942).

5.10 **Ears Help Eyes Spot Planes.** Pop. Sci. **141**, 123 (Oct. 1942).

5.10 **Distance Determination by Means of Sound.** D. Elkin and R. Tagamlitzkaya. Z. usn. Bol. **16**, 195–200 (1939).

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5.16 **Phonograph Needle (new "Jensen Concert Needle" with a Spring Action Construction).** Electronics **15**, No. 11, 132 (Nov. 1942).

5.16 **Can Sound-on-Film Supplant the Record?** R. H. Cricks. Electronic Engng. **14**, 708–710, 726 (Apr. 1942); Wireless Engineer **19**, Abs. 2687 (1942).

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5.16 **A Modern Music Recording Studio.** M. Rettinger. J. Soc. Mot. Pic. Eng. **39**, 186–194 (Sept. 1942).

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5.16 **Motion Picture Laboratory Practices.** J. R. Wilkinson. J. Soc. Mot. Pic. Eng. **39**, 166–184 (Sept. 1942).

5.16 **Foundations of Stereophonic Reproduction for Sound-Films, in which the Conditions in the Recording Studio are Reproduced in the Auditorium.** H. Warncke. E.T.Z. **63**, 244–245 (May 21, 1942).

5.17 **A New Telephone Set for the Hard of Hearing.** Alfred Herckmans. Bell Lab. Rec. **21**, No. 2, 45–48 (Oct. 1942).

6. Musical Instruments and Music

6.1 **Easy-to-Read Music.** Sci. Digest **12**, 96 (Oct. 1942).

6.1 **Simplification of Musical Notation.** W. E. Ingerson. Nature **150**, 323 (Sept. 12, 1942).

6.1 **Modern Science and Musical Theory.** Llewelyn S. Lloyd. J. Roy. Soc. Arts. **90**, No. 4619, 581–593 (Aug. 7, 1942).

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6.1 **Cooperation with the Film Industries in the Study of Primitive Music.** Carl E. Seashore. Science **96**, 263–265 (Sept. 18, 1942).

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6.1 **Woods Used in Musical Instruments.** A. L. Snoddy. Etude **60**, 584 (Sept. 1942).

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7. Noise

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7.7 **Vibration and Rubber Springs.** W. C. Keys. Mech. Engng. **64**, 175–180 (Mar. 1942); Sci. Abs. **A45**, 2058 (1942).

9. Speech and Singing

9.1 **Your Voice and the Telephone.** F. L. Hunt. Sci. Mon. **54**, 139–148 (1942).

10. Supersonics (Ultrasonics)

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10.5 **Propagation of Ultrasonic Waves in Liquid Mixtures and Intermolecular Forces, II.** Ram Prasad. Indian J. Phys. **16**, 1–11 (Feb. 1942).

10.5 **Propagation of Sound in Liquids, and Viscosity.** G. Suryan. Indian J. Physics **16**, 77–81 (Apr. 1942).

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10.7 **Influence des Ondes Ultrasonores sur la Diffraction des Rayons X par les Cristaux.** J. WEIGLE. Helv. Phys. Acta **15**, 329 (1942).

10.9 **The Influence of Ultra-Sounds on the Ear of Animals.** K. A. DERNNOVA AND A. I. TITOV. Z. usn. Bol. **16**, 104–113 (1939).

11. WAVES AND VIBRATIONS

11.1 **The "O.M. Corbino" National Electro-Acoustical Institute.** A. GIACOMINI. La Ricerca Scient. **13**, 249–266 (June/July, 1942); Wireless Engineer **19**, Abs. 3028 (1942).

11.3 **The Rate of Vibration of Plastics.** Science **96**, No. 2486, Suppl. p. 9 (Aug. 21, 1942).

11.4 **Power and Realism: Estimating the Watts Required for a Given Sound Intensity.** G. E. MORISON. Wireless World **48**, 130–132 (June, 1942).

11.4 **On the Calculation of the Sound Field Immediately in Front of a Circular Piston Diaphragm.** H. STENZEL. Ann. der Physik **41**, 245–260 (Apr. 29, 1942); Wireless Engineer **19**, Abs. 2707 (1942).

11.7 **The Vibrating Characteristics of Nearly-Complete, "Free-Free" Circular Rings.** F. P. BUNDY AND C. W. BANKES. J. Appl. Phys. **13**, 652–662 (Oct. 1942).

11.7 **The Transverse Vibrations of a Helical Spring with Pinned Ends and No Axial Load.** J. DICK. Phil. Mag. **33**, 513–519 (July, 1942).

11.7 **Vibration and Rubber Springs.** W. C. KEYS. Mech. Engng. **64**, 175–180 (Mar. 1942); Sci. Abs. **A45**, 2058 (1942).

11.7 **On a Generalization of Kirchhoff's Theory of Transversal Plate Vibrations in the Vibration Problem of Steam Turbine Disks.** I. MALKIN. Frank. Inst. J. **234**, 355–369 (Oct. 1942).

11.7 **The Application of the Laplace Transformation to a Problem on Elastic Vibrations.** C. J. TRANTER. Phil. Mag. **33**, 614–622 (Aug. 1942).

11.7 **On the Theory of Phase Compensation.** M. WALD. E.N.T. 19, 1–7 (Jan./Feb., 1942).

Review of Acoustical Patents

ROBERT W. YOUNG
C. G. Conn, Ltd., Elkhart, Indiana

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The decimal numbers appearing before titles are the numbers which identify the subject headings in the Cumulative Index of this journal issued in November, 1939. The Patent Office classification given in parentheses with each patent entry is outlined in the "Manual of Classification of Patents" which is obtainable from the Superintendent of Documents for $1.50.

Unless otherwise indicated, patents listed have been issued by the United States Patent Office.

Reviewers

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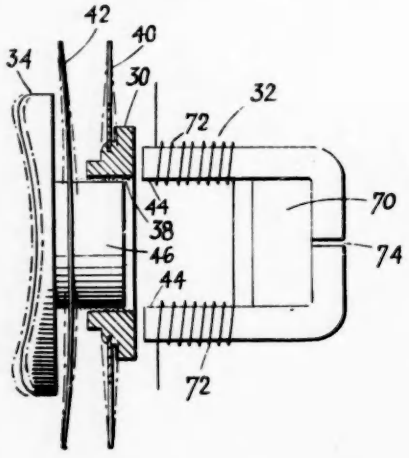
C. E. NELSON, *Nelson Muffler Corporation, Stoughton, Wisconsin*
R. W. YOUNG, *C. G. Conn, Ltd., Elkhart, Indiana*

2,291,942

4.5 BONE CONDUCTION HEARING AID

Samuel J. A. M. Bagno, assignor to Kurman Electric Company, Inc.
August 4, 1942, 10 Claims (Cl. 179–107).

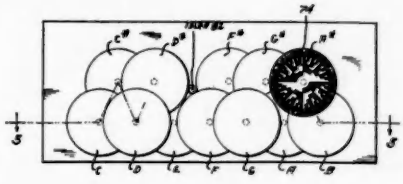


To prevent "freezing" of armature under steady pressure on contact button, this receiver has button (34) driven through the means of a viscous medium (heavy grease or the like) contained in the 0.002" clearance (38) between button plunger (46) and hole in armature (30).—B.B.B.

2,286,030

5.7 STROBOSCOPE

R. W. Young and Allen Loomis, assignors to C. G. Conn, Ltd.
June 9, 1942, 35 Claims (Cl. 88–14).

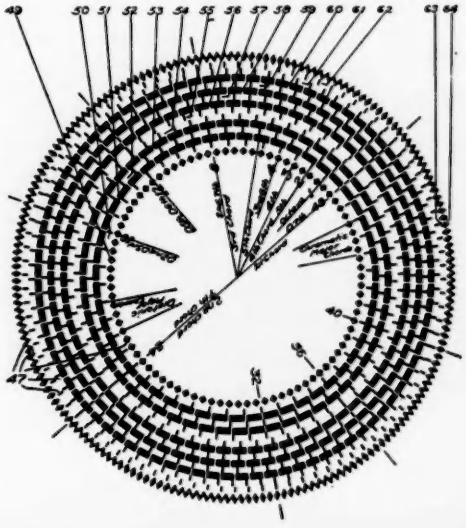


In this invention the stroboscopic effect is employed to indicate synchronism between the frequency of a sound and the rate of rotation of one of a series of interconnected disks, the number of which is related to the pattern inscribed thereon. In the embodiment illustrated by the figure, the disk pattern consists of 7 rings based on multiples of two, and the 12 disks have successive speeds whose ratio approximates the twelfth root of 2. This is accomplished by use, in alternation, of the gear ratios 89/84 and 107/101. The basic driving speed of the disks may be varied over a range corresponding to the twelfth root of 2, thereby permitting frequency measurements directly on a logarithmic scale over a continuous range of seven octaves. —R.W.Y.

2,296,777

5.7 TONOSCOPE

Carey F. Elton.
September 22, 1942, 8 Claims (Cl. 88–14).



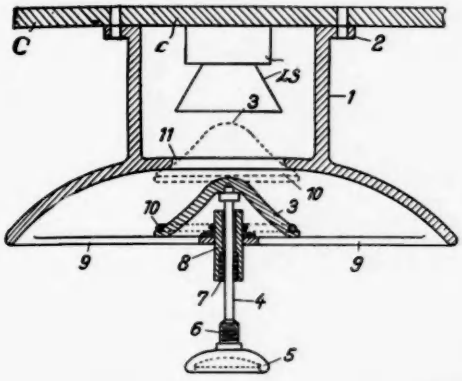
This stroboscopic tuning device has a rotating disk or drum on which patterns are marked which in use are intended to simulate the piano keyboard, relatively opaque sections corresponding to the "black" keys and translucent sections to the "white" keys. The composite pattern marked 63 and 64 consists of 185 and 196 high points, respectively, by which successive equally tempered semitones may be tuned upon utilizing a self-contained speed adjustment.—R.W.Y.

2,290,968

5.8 LOUDSPEAKER

Leopold Igluck, Berlin, Germany, assignor to C. Lorenz Aktiengesellschaft.

July 28, 1942, 2 Claims (Cl. 181–31).



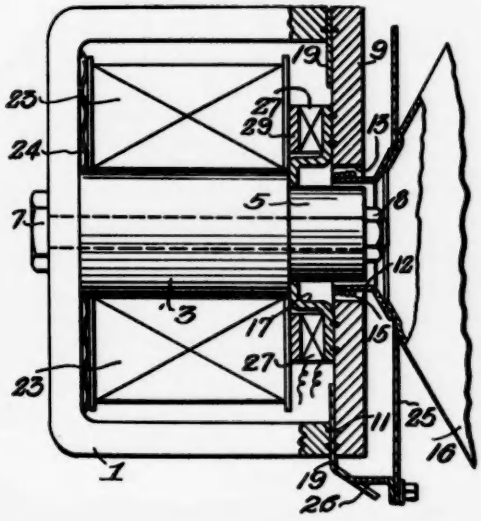
This patent discloses means for closing off loudspeakers used in humid or moist locations against the action of water.—H.S.K.

2,286,123

5.8 ELECTROMAGNETIC TRANSDUCER

Wm. F. Steers, assignor to Radio Corporation of America.

June 9, 1942, 3 Claims (Cl. 179–115.5).



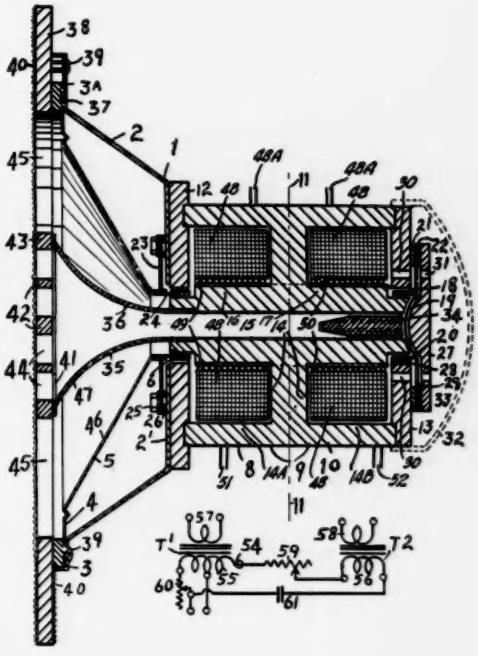
Hum reduction in field coil type loudspeakers has been attained to a degree about 2 db greater than the sum of the reductions of hum-bucking coil and shading ring used individually, by the proper use of these elements simultaneously. In addition, the reduction is more uniform with frequency.—H.S.K.

2,295,527

5.8 ELECTRODYNAMIC AND ELECTRO-ACOUSTICAL APPARATUS

Raymond E. Bowley, decree of distribution to Eva Mary Bowley.

September 15, 1942, 2 Claims (Cl. 179–116).



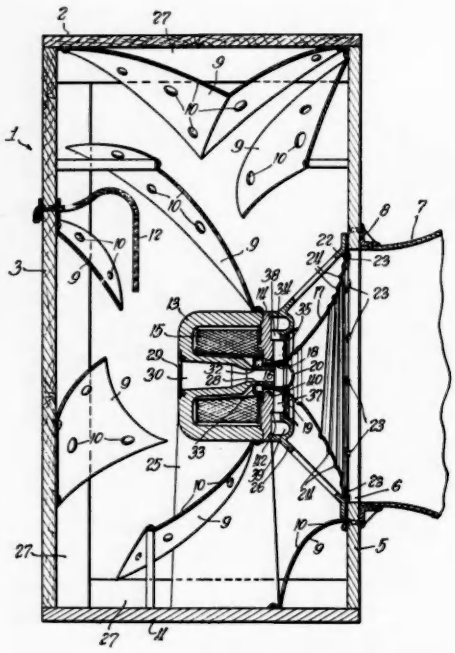
Of the two magnet structures on a common core described in this patent, one is used to drive a small diaphragm firing through the core and into a horn coplanar with the large end of the cone driven at the other end of the magnetic structure. The two acoustic structures are intended to handle complementary regions of the frequency spectrum.—H.S.K.

2,295,483

5.8 LOUDSPEAKER

Hugh S. Knowles, assignor to Jensen Radio Manufacturing Company.

September 8, 1942, 7 Claims (Cl. 179–115.5).

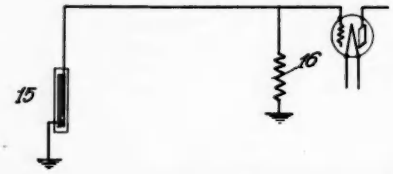


Among the features of the direct radiator loudspeaker referred to in this patent are damping and eccentric corrugation of the centering member at the small end of the cone; eccentric mounting of the speaker in an enclosure; the use of vanes for breaking up standing waves in the enclosure; and means for conducting heat out of the enclosure.—H.S.K.

2,284,039

5.9 REPRODUCTION OF SOUND

Wm. A. Bruno, assignor by mesne assignments to Bruno Patents Inc.
May 26, 1942, 4 Claims (Cl. 179–111).



The electrostatic field of an electret (part 3) is used in this capacity-operated transducer intended mainly as a microphone. A thin metallic diaphragm (part 5) is held in place by electrostatic forces, and its motion is restricted to spaces between high spots on the electret surface. Frequency response is controlled by perforations in the electret and protecting case.—H.S.K.

2,287,950

5.9 PIEZOELECTRIC DEVICE

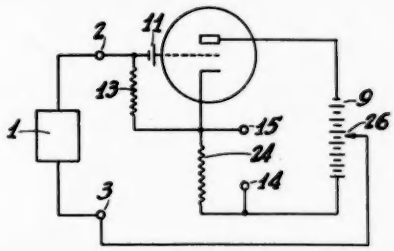
Raymond W. Tibbets.
June 30, 1942, 15 Claims (Cl. 171–327).

"A method of making piezoelectric devices comprising bodies formed of homogeneous water soluble crystalline material of high specific inductive capacity and having electrodes on one or more surfaces thereof, said method comprising applying directly to the body surface to be electroded a thin coating of water or other solvent of the body material, or a solution of the same material in a suitable solvent and applying under slight pressure a very thin metallic foil, such as gold leaf, having the property of forming with the recrystallized surface of the body material a molecular and intimate bond."—R.W.Y.

2,288,600

5.9 ELECTRICAL DEVICE

John P. Arndt, Jr., assignor to The Brush Development Company.
July 7, 1942, 12 Claims (Cl. 179–1).

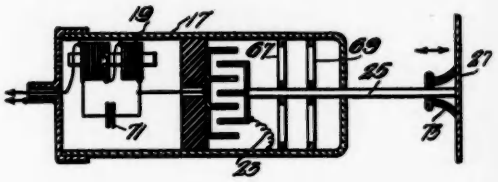


Amplifier for use with devices requiring high input impedance. Piezoelectric microphone is connected to the input terminals (2) and (3). Cathode follower output voltage across resistor (24) is fed back into grid through resistor (13). Since this voltage is substantially identical to the input voltage, the current in resistor (13) is practically zero and apparent input impedance is extremely high. Microphone return may be to a tap at the same d.c. potential as terminal (15).—B.B.B.

2,288,838

5.9 VIBRATION PICK-UP DEVICE

Eugene W. Pike and Arthur W. Vance, assignors to Radio Corporation of America.
July 7, 1942, 7 Claims (Cl. 177–351).



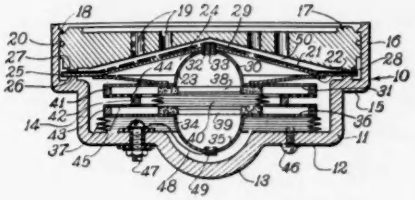
Pick-up for measurement of motion of bodies, having an inductance and a condenser connected in series and resonated at f_0. Either the condenser or the inductance is made to change in accordance with the body motion, thus modulating the current supplied from an external oscillator at or near f_0. Vibration is determined by reading (*a*) the voltage developed across one of the reactive elements, (*b*) the current in the series circuit, or (*c*) the relative phase of input voltage to input current.—B.B.B.

2,288,839

5.9 SOUND TRANSLATING DEVICE

Harold C. Pye, assignor to Associated Electric Laboratories, Inc.

July 7, 1942, 4 Claims (Cl. 179–122).



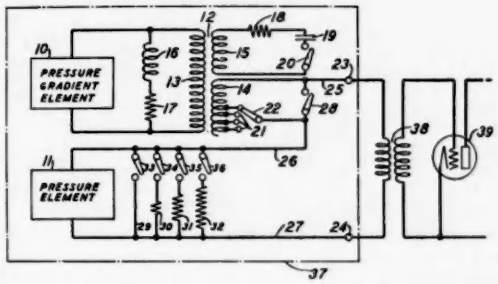
A microphone for elimination of low frequency noises, composed of two carbon electrode rings in contact with carbon granules (not shown). Between the rings is interposed a mechanical band-pass filter designed to prevent compression of carbon granules at low frequencies but permitting compression at high frequencies.—B.B.B.

2,293,258

5.9 ACOUSTIC DEVICE

William R. Harry, assignor to Bell Telephone Laboratories, Inc.

August 18, 1942, 1 Claim (Cl. 179–1).



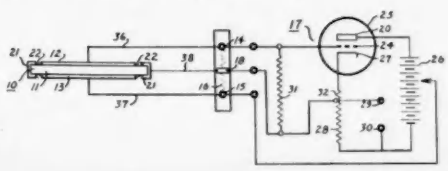
Transmitter, consisting of a velocity microphone connected in series with a pressure microphone, in combination with shunt resistors and switching means for varying the output of each microphone for the purpose of altering the over-all directional characteristic of the transmitter. —B.B.B.

2,289,954

5.9 LEAKAGE REDUCING MEANS

John P. Arndt, Jr., assignor to The Brush Development Company.

July 14, 1942, 19 Claims (Cl. 179–1).



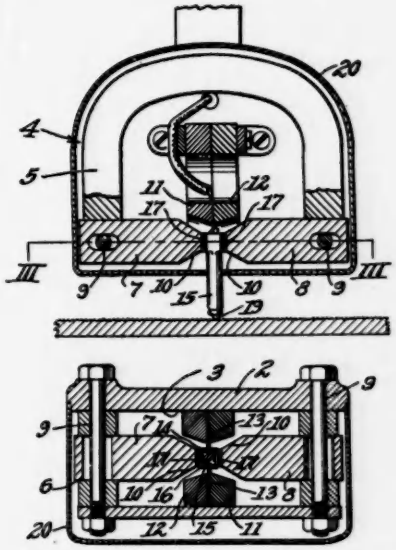
Apparatus for voiding the effect of d.c. surface leakage in piezoelectric crystals and for increasing input impedance of amplifiers. Piezoelectric microphone is connected to the input terminals (14) and (15). Cathode follower output voltage across resistor (28) is fed back to the "guard" electrode (21) and to the grid (24) (through resistor 31). Since this voltage is substantially identical to the input voltage, the current in resistor (31) and in the surface leakage resistance (22) is extremely small. The apparent input impedance is therefore extremely high. —B.B.B.

2,290,057

5.16 SOUND RECORDING AND REPRODUCING MECHANISM

Theodore Lindenberg, Jr.

July 14, 1942, 8 Claims (Cl. 179–100.41).



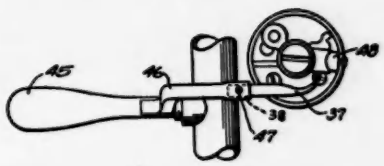
Pick-up consisting of tube (15) with reproducing stylus (19) attached at one end and square coil of wire (14) wound on the other end. Coil is held between pole-pieces by means of fibrous vanes (13) and two damping pads (17).—B.B.B.

2,284,016

6.3 WIND MUSICAL INSTRUMENT

Norman C. Pickering, assignor to C. G. Conn, Ltd.

May 26, 1942, 12 Claims (Cl. 84–390).



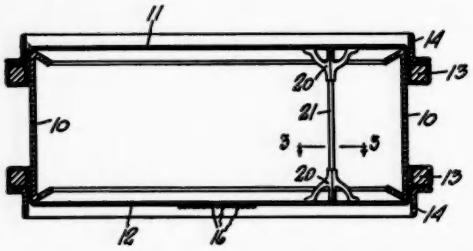
A spring constitutes the linkage between the finger lever and the valve rotor of a wind instrument. In addition to

supplanting the string often used in this position, the spring may be so bent that it has a tendency to curl toward the valve axis, and thereby increasing the torque tending to rotate the valve as the finger lever (45) is depressed.—R.W.Y.

2,285,269

6.4 DRUM VITALIZER

Joseph M. Grolimund, assignor to H. and A. Selmer, Inc.
June 2, 1942, 14 Claims (Cl. 84–415).

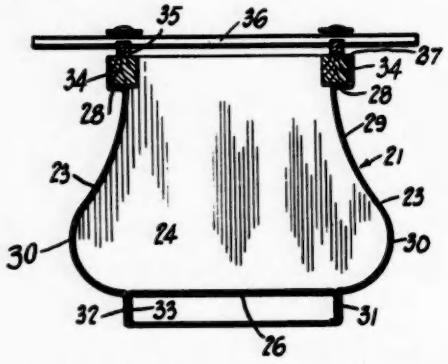


It is the purpose of this invention to "vitalize the tonal depth of a drum" by means of a rigid mechanical connection between the heads, contact being made through feet of relatively small area.—R.W.Y.

2,287,138

6.4 XYLOPHONE

Elmer Sas.
June 23, 1942, 7 Claims (Cl. 84–410).

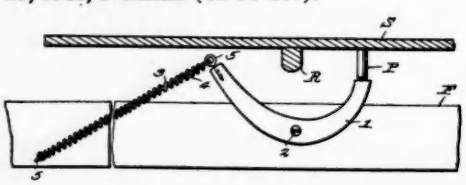


This xylophone is characterized by having only a single resonator, the sectional shape of which is shown in the figure, above whose open top a number of bars are mounted.—R.W.Y.

2,287,503

6.6 PIANO

Archie Mitchell Tyson.
June 23, 1942, 3 Claims (Cl. 84–189).

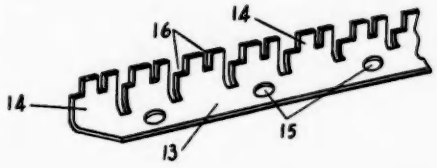


This invention relates to a means of producing a "singing" tone on piano, consisting of spring (3) which is vibrated by the movement of the sound board *S* through a pin and lever arrangement.—R.W.Y.

2,296,698

6.6 PIANO STRING SUPPORT

Hugo Benioff, assignor to Baldwin Piano Company.
September 22, 1942, 7 Claims (Cl. 84–211).

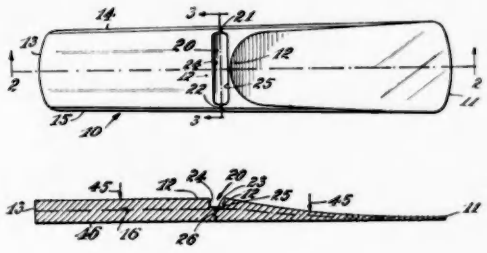


A support for piano strings at the "fixed end," flexible with respect to longitudinal motion, but rigid to transverse motion. It is claimed that longitudinal vibrations of the string contribute to the clang sound, particularly of bass strings, and that the above-mentioned flexibility minimizes this disagreeable sound. The flexibility is gained by a sheet metal agraffe (see figure) for upright pianos and a thinned agraffe for grands.—R.W.Y.

2,287,529

6.8 REED OF CANE, PLASTIC, OR ANY OTHER MATERIAL FOR CLARINETS, SAXOPHONES, AND LIKE MUSICAL INSTRUMENTS

Mario Maccaferri.
June 23, 1942, 1 Claim (Cl. 84–383).

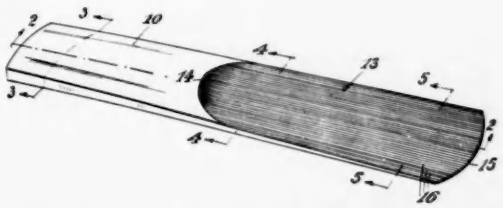


The object of this invention is to provide a reed shape such as to make more brilliant the tone quality resulting from its vibration. The result is produced by cutting notches within the center sixth of the reed, but not extending entirely across nor deeper than the median plane.—R.W.Y.

2,296,737

6.8 REED

Walter M. Peterson, assignor to Wm. R. Gratz Company, Inc.
September 22, 1942, 3 Claims (Cl. 84–383).

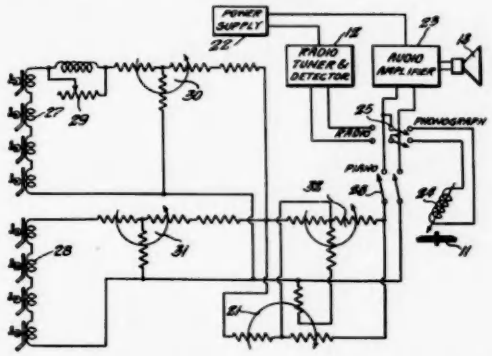


"The object of this invention is to provide a reed of plastic material which will have a fineness at the tip to respond to the most delicate tones intended to be played, but to have such strength longitudinally as to resist longitudinal buckling." The desired lightness with rigidity is obtained by longitudinal "rills."—R.W.Y.

2,285,132

6.9 COMBINATION ELECTRIC MUSICAL INSTRUMENT

Paul Weathers et al., assignors to Radio Corporation of America.
June 2, 1942, 3 Claims (Cl. 84–127).

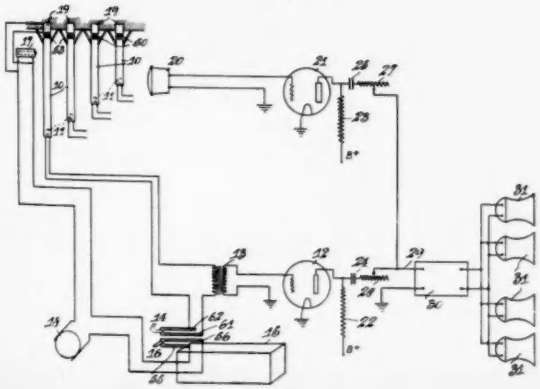


Electronic piano having two volume controls operating on the same channel, one control being actuated manually, the other by a pedal.—R.C.F.

2,286,587

6.9 CARILLON

E. V. Sundt.
June 16, 1942, 14 Claims (Cl. 84–1.14).

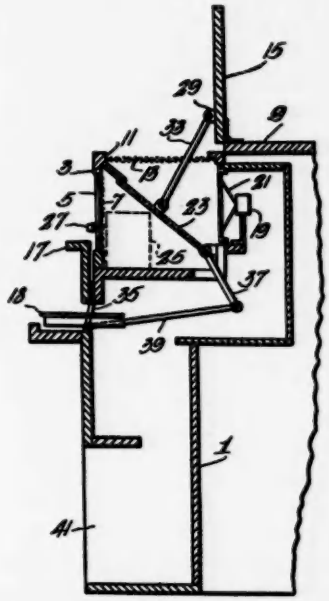


Chime carillon having electrical pick-up and electronic means for controlling attack and decay of tone.—R.C.F.

2,286,742

6.9 SOUND-PRODUCING AND SOUND-REPRODUCING APPARATUS

E. W. Kellogg, assignor to Radio Corporation of America.
June 16, 1942, 9 Claims (Cl. 84–1.01).

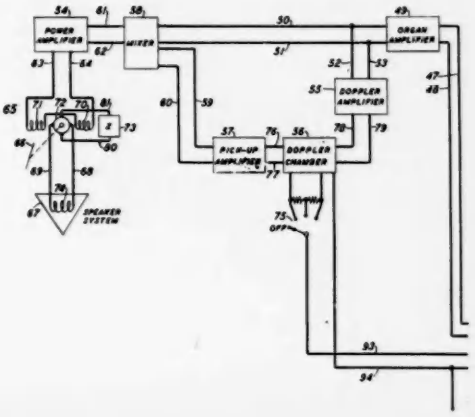


Combination electronic piano and radio receiver or phonograph, having mechanical means for rendering sound radiation from speaker direct or indirect, according to the source from which it is energized.—R.C.F.

2,287,105

6.9 ELECTRONIC ORGAN

W. F. Kannenberg, assignor to Bell Telephone Laboratories, Inc.
June 23, 1942, 13 Claims (Cl. 84–1.21).

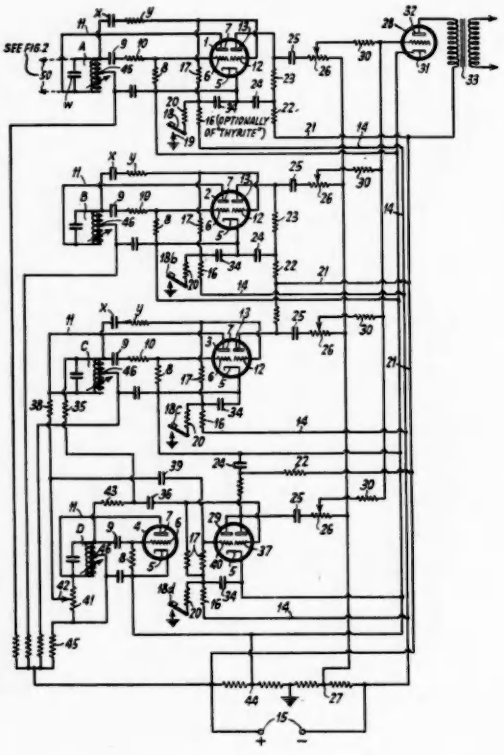


Harmonic control means comprising bridge network; swell control means having phase-shifting network; choral effect means comprising acoustic chamber.—R.C.F.

2,292,757

6.9 ELECTRONIC CHIME SYSTEM

Jarrett L. Hathaway, assignor to Radio Corporation of America.
August 11, 1942, 11 Claims (Cl. 84–1.26).

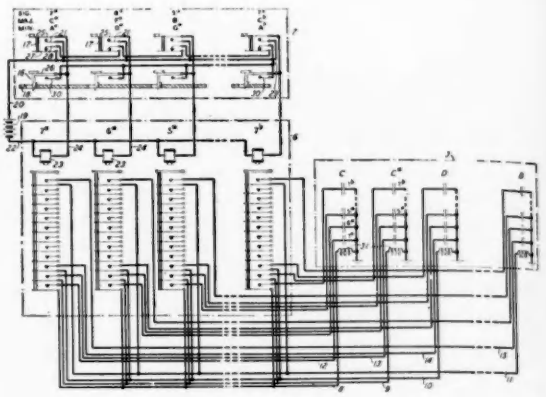


Vacuum tube oscillator and amplifier circuits adapted to simulate the attack and decay characteristics typical of acoustic chimes.—R.C.F.

2,293,499

6.9 MUSICAL INSTRUMENT

Sidney T. Fisher, assignor to Western Electric Company, Inc.
August 18, 1942, 9 Claims (Cl. 84–1.11).

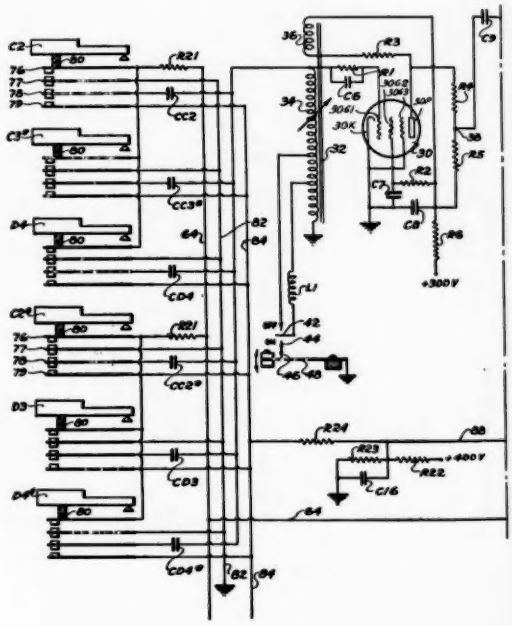


Switching means in an instrument of vacuum-tube oscillator type to permit playing in just intonation in any key. —R.C.F.

2,294,178

6.9 ELECTRICAL MUSICAL INSTRUMENT

J. M. Hanert, assignor to Hammond Instrument Company.
August 25, 1942, 14 Claims (Cl. 84–1.20).

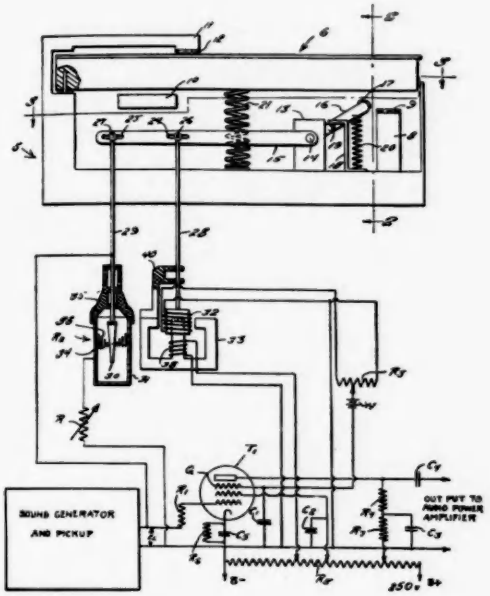


A musical instrument of melody (Solovox) type, having key-controlled contacts to over-bias and "paralyze" an amplifier tube when more than one key is simultaneously pressed, preventing the sounding of false pitches; also transient-suppressing means.—R.C.F.

2,296,125

6.9 AUTOMATIC VOLUME CONTROL FOR KEYBOARD OPERATED MUSICAL INSTRUMENTS

John L. Traub.
September 15, 1942, 5 Claims (Cl. 84–1.27).

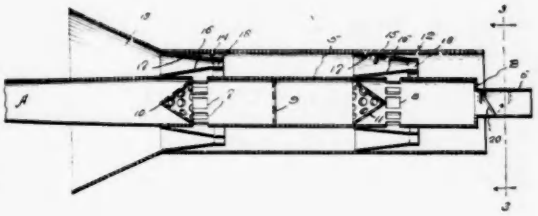


Key-operated means for controlling the volume of individual tones in accordance with the depth of depression of the keys.—R.C.F.

2,285,674

7.7 MUFFLER OR SILENCER

Cecil L. Merett.
June 9, 1942, 1 Claim (Cl. 181–51).

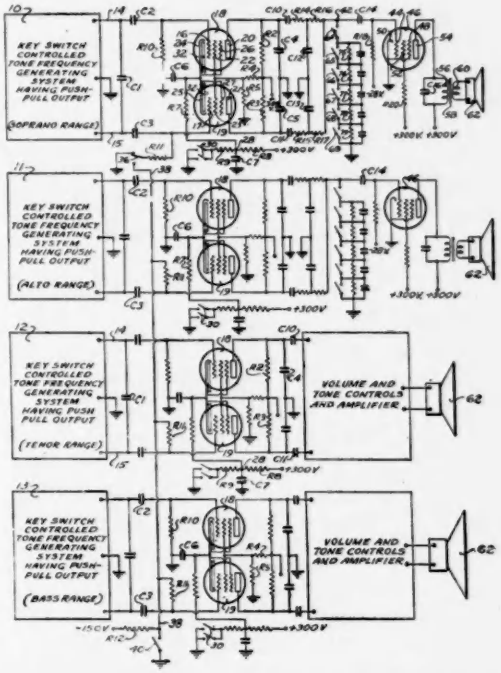


An exhaust silencer incorporating a Venturi unit which claims to reduce engine back-pressure and silence the engine exhaust.—C.E.N.

2,295,524

6.9 ELECTRICAL MUSICAL INSTRUMENT

John M. Hanert, assignor to Hammond Instrument Company.
September 8, 1942, 8 Claims (Cl. 84–1.19).

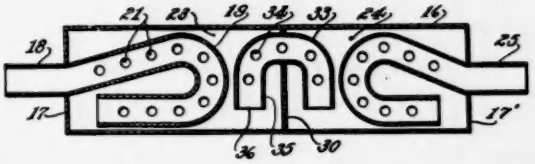


A musical instrument having a plurality of channels for sounding separately the several parts of harmonic music, in which unbalanced push-pull-connected variable-mu tubes are used in each channel for harmonic production.—R.C.F.

2,290,818

7.7 SILENCER

John P. Tyskewicz, assignor to The Maxim Silencer Company.
July 21, 1942, 14 Claims (Cl. 181–53).



An exhaust silencer in which the inlet and/or outlet tubes are arranged so as to reverse completely the direction of gas flow one or more times. Conduits have lateral escape openings in their walls.—C.E.N.